Study of Fuzzy Set Theory and Multi Criteria Decision Making Problems

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Abstract:

Multi Criteria Decision Management (MCDM) attempts to solve problems in a dynamic decision-making setting, which is a crucial challenge in a variety of applications including product design, service provider selection, and quality selection. In the literature, there are some effective strategies for determining the best variety based on the criterion. Due to increased costs, needless data entry, poor product consistency, and time waste, these approaches struggle to achieve high selection accuracy. As a result, enhanced Hierarchical-Fuzzy (H-Fuzzy) set theory is used to solve MCDM problems in product design applications in this proposed methodology. Using normalised average weight gain operation, the proposed H-Fuzzy theory is used to choose the best product tool. Priority weighting and normalised weighting are the two key stages in the proposed H-Fuzzy set theory. In H-Fuzzy theory, priority weighting is achieved, and the total priority weights alternatives are calculated. The alternatives are rated based on these overall priority weights.

Keywords:

Criteria, Alternative, H-Fuzzy, Accuracy, product design, MCDM problems, weighting

1. INTRODUCTION

Multi-Criteria Decision making (MCDM) is one of the fastest growing problem. It is applicable in the field of business, product design, software tool selection and quality selection [1]. A MCDM problem varies from single criterion decision environment to multi criteria decisions. These decision making approach is implemented in realistic environments. Many methods have been proposed to make appropriate decisions. Two main streams are developed in this approach, such as multi objective decision making and multi attribute decision making. Multi objective decision making approach determines the optimal solution by assuming that any problems can be modelled as mathematical one. A continuous calculation of the problem mathematically gives optimal solution. Multi attribute decision making [2] handles problems as discrete space alternatives. Discrete mathematical calculations of the problem give decisions in this approach. Ranking the decision alternatives among the existing alternatives is the method utilized in these models to make decisions. The measurement process for modelling problems consists in the construction of scales by mapping or transforming empirical results into numerical ones in such a way that the information is preserved. Even though this type of problem is much more relevant and frequent in practice, there are MCDM methods available to solve MCDM problems and their quality is much harder to determine.

MCDM may be considered as a complex and dynamic process including one managerial level and one engineering level. The managerial level defines the goals, and chooses the final optimal alternative. The multi criteria nature of decisions is emphasized at this managerial level, at which public officials called decision makers have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are off line from the optimization procedure done at the engineering level. Very often, the preference structure is based on political rather than only technical criteria. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of non-inferior and/or compromise solutions. The engineering level of the MCDM process defines alternatives and points out the consequences of choosing any one of them from the standpoint of various criteria. This level also performs the multi criteria ranking of alternatives.

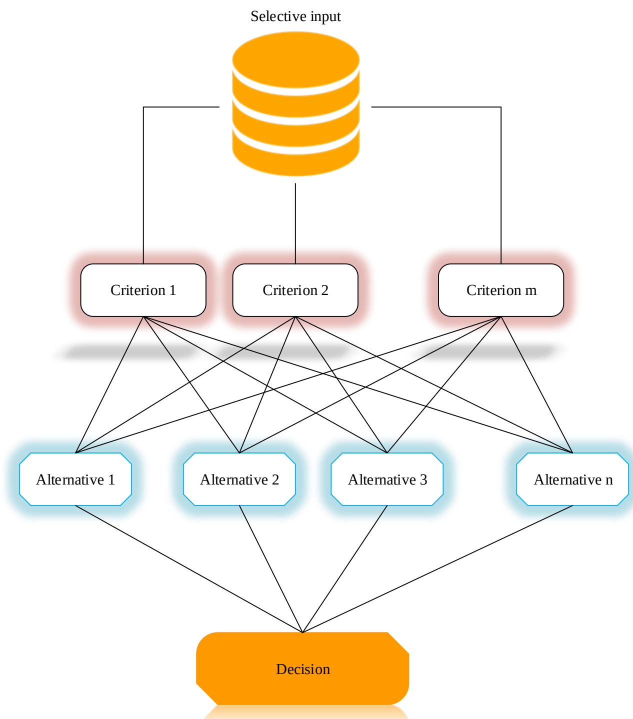

Figure 1: MCDM approach

The decision making approach includes multi set of decision alternatives for selection process. In the decision making process, set theory is used to selection process which considered as main progress to enable proper decisions related to problems. Generally, different types of methods are available to solve MCDM issues by enable the proper decisions such as AHP, Analytic Network Process (ANP) set theory and Fuzzy Set theory. TOPSIS method utilizes high degree of distance method to compare which neglects the alternatives accordingly. However, this method provides the inappropriate decisions. Similarly, AHP is used to solve the MCDM problems by provides the best decisions but it has a weak decision making solution and it trap to identify and weight of designing problem. Additionally, ANP is unable to provide perfect ranking alternatives. And, fuzzy set theory permits the combination of qualitative and quantitative with a partially known data into a decision making process.

Previously, a multi-criteria decision making framework for location planning for urban distribution centres under a fuzzy environment. This approach comprises four steps. In step 1, we identify the criteria for evaluating potential locations for distribution centres. These criteria are: Accessibility, Security, Connectivity to multimodal transport, Costs, Environmental impact, Proximity to customers, Proximity to suppliers, Resource availability, Conformance to sustainable freight regulations, Possibility of expansion, and Quality of service. In step 2, the potential locations for implementing urban distribution centres are identified. In step 3, the decision makers provide ratings for the criteria and the potential locations. Fuzzy TOPSIS is used to determine aggregate scores for all potential locations and the one with the highest score is finally chosen for implementation. Sensitivity analysis is performed to assess the influence of criteria weights on the decision making process. Therefore, we can say that the location decision is relatively insensitive to benefit criteria weights; however when the weights of cost criteria (C3, C4) are set as the highest, then the best solution is changed from A1 to A3.

In this fuzzy set theory decisions are made to quantify the uncertainty and to handle the partial data involved in the process of decision making. This theory allows mathematical operators and the programming approach to apply in the fuzzy system model. The fuzzy set theory may fails to obtain the appropriate solution in complex decision making process. The abovementioned drawbacks are motivated by design a best method to enable the proper decision in MCDM problems.

1.1 Basic Architecture of Fuzzy Logic System

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made, or patterns discerned [7]. In fuzzy based system five major process will be carried out. The schematic representation of architecture of fuzzy logic system was illustrated in figure 2

The process of fuzzy inference involves all the pieces that are described in Membership Functions, Logical Operations, and If-Then Rules. Following that the stepwise procedure for process carried out in fuzzy is given below.

- ❖ Fuzzification of input variable
- ❖ Application of fuzzy operator (AND or OR) in the antecedent
- ❖ Implication from the antecedent to the consequent
- ❖ Aggregation of the consequent across the rules
- ❖ Defuzzification

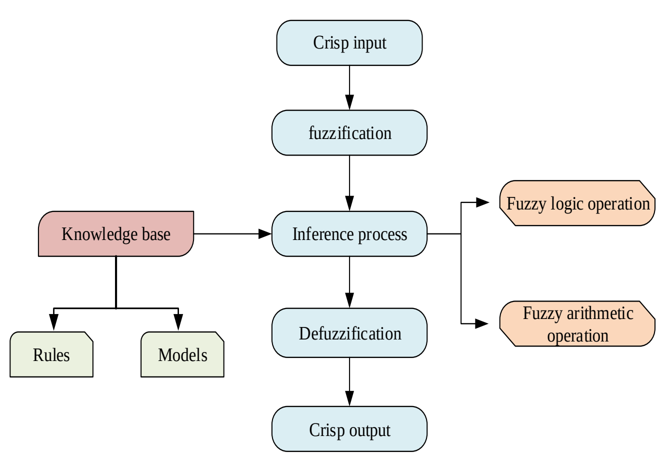

Figure 2: Basic Architecture of Fuzzy Logic System

a) Fuzzy input

The first step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions (Fuzzification). In Fuzzy Logic Toolbox software, the input is always a crisp numerical value limited to the universe of discourse of the input variable (in this case, the interval from 0 through 10) . The output is a fuzzy degree of membership in the qualifying linguistic set (always the interval from 0 through 1). Fuzzification of the input amounts to either a table lookup or a function evaluation.

b) Apply fuzzy operator

After the inputs are fuzzified, you know the degree to which each part of the antecedent is satisfied for each rule. If the antecedent of a rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the rule antecedent. This number is then applied to the output function. The input to the fuzzy operator is two or more membership values from fuzzified input variables. The output is a single truth value. As is described in Logical Operations section, any number of well-defined methods can fill in for the AND operation or the OR operation. In the toolbox, two built-in AND methods are supported: min (minimum) and prod (product).

c) Apply implication method

Before applying the implication method, you must determine the rule weight. Every rule has a weight (a number from 0 through 1), which is applied to the number given by the antecedent. Generally, this weight is 1 (as it is for this example) and thus has no effect on the implication process. However, you can decrease the effect of one rule relative to the others by changing its weight value to something other than 1.

After proper weighting has been assigned to each rule, the implication method is implemented. A consequent is a fuzzy set represented by a membership function, which weights appropriately the linguistic characteristics that are attributed to it. The consequent is reshaped using a function associated with the antecedent (a single number). The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Implication is implemented for each rule. Two built-in methods are supported, and they are the same functions that are used by the AND method: min (minimum), which truncates the output fuzzy set, and prod (product), which scales the output fuzzy set.

d) Aggregate all outputs

Since decisions are based on testing all the rules in a FIS, the rule outputs must be combined in some manner. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Aggregation only occurs once for each output variable, which is before the final Defuzzification step. The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule. The output of the aggregation process is one fuzzy set for each output variable.

As long as the aggregation method is commutative, then the order in which the rules are executed is unimportant. Three built-in methods are supported:

- ❖ max (maximum)
- ❖ probor (probabilistic OR)
- ❖ sum (sum of the rule output sets)

e) defuzzify

The input for the Defuzzification process is a fuzzy set (the aggregate output fuzzy set) and the output is a single number. As much as fuzziness helps the rule evaluation during the intermediate steps, the final desired output for each variable is generally a single number. However, the aggregate of a fuzzy set encompasses a range of output values, and so must be defuzzified to obtain a single output value from the set. There are five built-in Defuzzification methods supported: centroid, bisector, middle of maximum (the average of the maximum value of the output set), largest of maximum, and smallest of maximum.

The remaining part of the paper is organized as follows, section 2 briefs on the literature review in which the previously existing techniques related to solve MCDM problems and section 3 presents a detailed description in the proposed methodology related to selection of product.. Finally, section 4 briefs the conclusion part of the research.

2. 2. LITERATURE REVIEW

Many different methods are designed by researchers to solve MCDM problems with various applications. Some of the methods are reviewed in this section which useful to identify the problem formulation of proposed methodology.

Liao *et al* have developed an AHP approach that can be used in a variety of fields. Fuzzy extensions of AHP have been used to deal with the inevitability of ambiguity in decision-making. This author looked into a hesitant fuzzy linguistic extension of AHP, which expanded the method's applications. This paper proposed algorithms for consistency testing and inconsistency repair. The interval method was used to avoid discretizing the Decision Maker's continuous semantic interval (DM). This method generated a collection of perfectly consistent reluctant fuzzy linguistic preference relations (HFLPR) that preserved as much as possible the initial DM evaluations. It's worth noting, however, that traditional AHP isn't built to capture ambiguous desires in human perceptions.

Nguyen *et al* have created ANP, which is used in the context of Intuitionistic Fuzzy Values (IFV). When MCDM issues were addressed, IFV was beneficial to DMs. The membership degree, non-membership degree, and hesitancy degree were all expressed in IFV, which almost showed the value that DMs produced. This method established a new priority determining method from IFV to avoid the complicated calculations that arise from the IFVs' multi-dimensional degrees. If the linguistic term's hesitation value is the same, the DM must choose between the two words or the tie must be broken arbitrarily.

Kumar *et al* have presented a type-2 fuzzy system-based Interval type two fuzzy logic system (IT2FLS)-based short-term load forecasting approach to cancer DNA microarray data classification. The use of FCM clustering revealed the parameters of fuzzy rules and thus improved the capability of IT2FLS. The estimation method is carried out using data from the years 2005 to 2011 in Makassar City, Indonesia, to estimate the peak load of the next holiday. The validation results revealed that this approach is capable of providing forecasting results, as evidenced by small absolute error (AE) of less than 2% on average for the estimation task for the years 2012 and 2013. Large rule-based systems with high computational costs were needed. This was one of the reasons why researchers haven't given fuzzy systems enough credit for successfully solving cancer gene expression analysis problems.

Choi *et al* For operational usability-security and their combination at the early stage of web application development, we created an AHP and TOPSIS. The life cycle development process necessitates clarifying current usability perceptions and identifying an existing security evaluation system. For functional security estimation, this paper used the fuzzy-based AHP-TOPSIS process. The most important factor discovered through the system and presented hierarchy was user error security. Protection reliability was the second most important feature. As a result, the findings confirm that a lengthy diagnosis in input parameters was needed to achieve usable-secure web application security durability.

Dursun *et al* had introduced a fuzzy multi-criteria community decision tool using the Fuzzy Weighted Average (FWA) framework. It is used to determine the upper and lower limits of the weighted supplier selection requirements and supplier scores. The upper and lower bounds of the weights of supplier selection parameters were calculated by applying FWA to the data in the first metrics, while the upper and lower bounds of the scores of suppliers were determined by using FWA as inputs in the second metrics. Since most fuzzy number ranking methods were seldom used in this case, the suppliers were ranked using a ranking system that was said to be more effective and reliable than its predecessors.

Shermeh *et al* had demonstrated a multi-criterion recommendation approach based on a customer's real-time web usage behaviour, which was automatically registered in a web usage database. The relative value of specification weights is defined in the form of inequalities by evaluating an individual's implicit preference decisions on a subset of the items. The weights calculated were then used to prioritise the items that were not included in the navigation behaviours. Due to factors such as time constraints, cognitive load, and so on. The prescriptive MCDM method's solutions are invariably too incomplete to solve the decision problem at hand. As a result, some abstractions were necessary to arrive at a suitable solution at the cost of precision.

3. PROBLEM STATEMENT

Basically, the process of decision making approach for solving various existing and upcoming problem in different field was found to be crucial. Many authors are focusing this area for their research because this process was found to be challenging task. Several technique utilizing different theories were developed by various authors. But in every theories there exist certain limitations. Theories such as approximate reasoning theory, vague set theory, rough set theory, probability theory, soft set theory were developed in recent year for solving various MCDM problems. The problem such as effective selection of equipment used in various industry, designing concept evaluation, selection of appropriate location for suiting military airport and other essential buildings, effective solid transportation etc. comes under MCDM methods. The above mentioned technique were developed for solving these prevailing problem.

There exist certain limitation in these developed theories such as shortage in parametrization of tools and these theory does not process effectively in defining the vague concepts. Prior to this for solving problem in various field, the decisions were obtained through certain experts on the basis of skill and knowledge acquired in that particular field. But this way of approach was found to be too complex and difficulties exist in reaching the suitable solution. As well as the obtained solution was not accurate. For solving these drawbacks prevailing in classical set theory, the fuzzy set theory was developed and utilized in various problem solving applications. And the fuzzy set possess unique advantage such as it possess the ability to generalize 0 and 1 membership values of a crisp set to a membership function of a fuzzy set. On considering these advantages the fuzzy set theory based approach will be developed in this proposed work.

4. OBJECTIVE OF RESEARCH WORK

1. The fuzzy set theory based technique will be developed in this present work for utilization in various problem solving approaches.
2. Modification will be employed in the conventional fuzzy set theory and improved version of fuzzy set theory will be designed.
3. The modified fuzzy set theory will be applied in normally existing MCDM problem such as boat problem, inverted pendulum problem, solid transportation problem etc.
4. The results will be obtained after implementation of this proposed system and compared with other existing methods.
5. The modified fuzzy set theory developed in this proposed work will be proved as an effective

technique for application in various real time problems.

6. Improved fuzzy set theory will be developed and solved one or more MCDM problem in a mathematical way.

5. CONCLUSION

Multi-criteria decision making (MCDM) is one of the well-known topics of decision making. Fuzzy logic provides a useful way to approach a MCDM problem. Very often in MCDM problems, data are imprecise and fuzzy. In a real-world decision situation, the application of the classic MCDM method may face serious practical constraints, because of the criteria containing imprecision or vagueness inherent in the information. For these cases, fuzzy multi-attribute decision making (MADM) and fuzzy multi-objective decision making (MODM) methods have been developed. In this chapter, crisp MADM and MODM methods are first summarized briefly and then the diffusion of the fuzzy set theory into these methods is explained.

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