MINOR RESEARCH PROJECT

Development of some Statistical Process Control Techniques
"A Nonparametric Group Runs Control Chart to Detect Shifts in the Process Median"

Final report submitted
to
The joint secretary and Head
University grants commission
Western Regional Office (WRO)
Ganeshkhind,
Pune-411 007

During XI Plan period
2011-2013
UGC Reference No. : 23-2473/10(WRO)

by
Principal Investigator
Prof. Vikas Chintaman Kakade.
Associate Professor of Statistics
Department of Statistics
Anekant Education Society's
Tuljaram Chaturchand College
Baramati – 413 102
Maharashtra

DECEMBER 2013

CERTIFICATE

This is to certify that the final report on UGC Minor research project entitled **"**A Nonparametric Group Runs Control Chart to Detect Shifts in the Process Median**"** is a record of bonafide research work carried out by Prof. Vikas Chintaman Kakade, Associate Professor of Statistics, Tuljaram Chaturchand College, Baramati, Maharashtra. A copy of the final report of Minor Research Project has been kept in the library of College and an executive summary of the report has been posted on the website of the College.

Principal

Dr. C. V. Murumkar.

Acknowledgement

I express the deep sense of gratitude to the Joint Secretary and Head WRO, UGC, Pune, for sanctioning the minor research project under XI Plan period.

I acknowledge with gratitude for the co-operation and guidance extended to me by Dr. M. P. Gadre, Associate Professor, Department of Statistics, University of Pune.

I extend my respectful thanks to the College Management A.E. Society's for their support and co- operation. I have great pleasure in expressing my heartfelt thanks to Principal Dr. C. V. Murumkar, for the kind support and encouragement.

I express my indebtedness to Prof. N.J. Subandh, HOD, Statistics Department and my all collogues in Statistics and Computer Science department for their scholastic approach and human simplicity. I wish to express my thanks to Prof. P. G. Dixit, Prof. V. Y. Pawar, Prof. A.S.Jagtap, Prof. N.K Dhane, Dr. V.V. Patil, Prof. Chandrashekhar Swamimathapati and Mr. Saurabh Kadam for their assistance and help. I also thanks to Mr. V.P. Morankar, BOSH, Jalgaon, he gives me a friendly advice, encourage me and a valuable tips.

I will fail in my duties if I won't express my heartfelt thanks to my wife, my children and Mrs. M. M. Gadre for their sacrifice and support throughout the Research work.

Place :

Date:

(Vikas C. Kakade)

Contents

A Nonparametric Group Runs Control Chart to Detect Shifts in the Process Median

ABSTRACT

In this article, we propose a nonparametric 'Group Runs' (GR) control chart to detect shifts in the process median based on the signed-rank statistic to monitor shifts in the known in-control process median. A nonparametric Shewhart-type synthetic control chart introduced by Pawar and Shirke (2010) is also based on the signed-rank statistic. It is verified that, a nonparametric GR control chart gives a significant reduction in out-of-control 'Average Time to Signal' (ATS) as compared to a nonparametric Shewhart-type synthetic control chart when in-control ATS is not smaller than a certain number.

It is also verified that, a nonparametric GR control chart gives a significant reduction in out-of-control 'Average Run Length' (ARL) as compared to a nonparametric Shewhart-type synthetic control chart for the same in-control ARL values for the three distributions considered here.

***Key words*:** $\overline{X}$ chart, CRL chart, Synthetic Chart, Average Time to Signal (ATS), Steady State ATS, Average Run Length (ARL).

Table List

1. Introduction

Statistical process control (SPC) refers to the collection of statistical procedures and problem-solving tools used to control and monitor the quality of the output of some production process, including the output of services. The aim of SPC is to detect and eliminate or, at least reduce, unwanted variation in the output of a process. The benefits include saving time, increasing profits and an overall increase in the quality of products and services. SPC has long been applied in high-volume manufacturing processes. There are primarily service industries where the "volume" or the "speed" of production is less in comparison to the usual manufacturing process and the quality characteristics are less tangible and not easily measured on a numerical scale. The key idea, however, is that the principles and concepts of SPC can be applied to any repetitive process, i.e. a process wherein the same action is performed over-and-over with the intention to obtain the same "outcome" or "result" on each "trial".

A wide range of statistical procedures are used in the various stages of SPC. The basic descriptive techniques and summary measures such as histograms, stem-and-leaf diagrams, check sheets, scatter diagrams etc. are applying in SPC. Also there are some advanced procedures such as process optimization, evolutionary operation and design of experiments applying in SPC technique. Many of the statistical procedures like acceptance sampling and sampling schemes, measurement systems analysis, calibration, process capability analysis and capability indices, reliability analysis, statistical and stochastic modeling, six sigma that are used in SPC.

Control chart

A control chart is a statistical procedure that can be depicted graphically for on-line process monitoring of a measurable characteristic (such as the mean measurement value or the percentage nonconforming items) with the objective to show whether the process is operating within the limits of expected variation. The simplest and most widely used control chart is the Shewhart-type of chart; this chart is named after the father of quality control i.e. Dr. Walter A. Shewhart (1891-1967) of Bell Telephone Laboratories, who developed the chart in the 1930's and laid the foundation of modem statistical process control in his book *Economic Control of Quality of Manufactured Product* that was originally published in 1931.The the wider use and popularity of control charts outside manufacturing, which lead to Quality Management and Six Sigma, can be attributed to Deming (1986).

Variables and Attributes control charts

A quality characteristic that can be measured on a numerical scale is called a variable. Examples include width, length, temperature, volume, speed etc. When monitoring a variable we need to monitor both its location (i.e. mean or average) and its spread (i.e. variance or standard deviation or range). Sample statistics most commonly used to monitor the location of a process are the sample mean and the sample median or some other percentile (order statistic), whereas the sample range, the sample standard deviation and the sample variance are regularly used to monitor the process variation.

In situations where it is not practical or the quality characteristics cannot conveniently be represented numerically, we typically classify each item as either conforming or nonconforming to the specifications on the particular quality characteristic(s) of interest; such types of quality characteristics are called attributes. Some examples of quality characteristics that are attributes, are the number of nonconforming parts manufactured during a given time period or the number of tears in a sheet of material.

The p-chart and the np-chart are attribute charts that are based on the binomial distribution and are used to monitor the proportion (fraction) of nonconforming items in a sample and the number of nonconforming items in a sample, respectively. Another type of attribute chart is the c-chart, which is based on the Poisson distribution, and is useful for monitoring the number of occurrences of nonconformities (defects) over some interval of time or area of opportunity, rather than the proportion of nonconforming items in a sample.

Parametric and Nonparametric control charts

In the process control environment of variables data (i.e. data that can be measured on a continuous numerical scale) parametric control charts are typically used; these charts are based on the assumption that the process output follows a specific distribution, for example, a normal distribution. Often this assumption cannot be verified or is not met. It is well-known that if the underlying process distribution is not normal, the control limits are no longer valid so that the performance of the parametric charts can be degraded. Such considerations provide reasons for the development and application of easy to use and more flexible and robust control charts that are not specifically designed under the assumption of normality or any other parametric distribution. Distribution-free or nonparametric control charts can serve this broader purpose.

A thorough review of the literature on nonparametric control charts can be found in Chakraborti et al. (2001, 2007). The term nonparametric is not intended to imply that there are no parameters involved, quite to the contrary. While the term distribution-free seems to be a better description of what one expects these charts to accomplish, nonparametric is perhaps the term more often used; in this thesis, both terms (distribution-free and nonparametric) are used since for our purposes they mean the same.

The main advantage of nonparametric charts is their general flexibility i.e. their application does not require knowledge of the specific probability distribution for the underlying process. In addition, nonparametric control charts are likely to share the robustness properties of the well-known nonparametric tests and confidence intervals; these properties entail, among others, that outliers and/or deviations from assumptions like symmetry far less impact them.

The run-length is defined as the number of samples to be collected or the number of points to be plotted on the chart before the first or next out-of-control signal is observed. The discrete random variable defining the run-length is called the *run-length random variable* and the distribution of this random variable is called the *run-length distribution.*

A formal definition of a nonparametric or distribution-free control chart could be given in terms of its run-length distribution, namely that, if the in-control run-length distribution is the same for every continuous probability distribution, the chart is called distribution-free or nonparametric (see e.g. Chakraborti et al. 2001, 2007).

2. Research objectives

Control charts are useful tools for monitoring a manufacturing process. Statistical process control is an effective technique useful to improve quality of the product. Most of the control charts are distribution-based, in the sense that the process output is assumed to follow a specified probability distribution like normal for continuous measurements. All the processes are not always normally distributed. In many practical situations, the standard control charts do not perform well for non-normal data. So there is demand for non-parametric control charts. A chart is said to be nonparametric, if the run length distribution of the chart does not depend on the underlying process distribution when there is no shift in the process parameter under study. Hence, in-control Average Run Length (ARL) of nonparametric chart does not depend on the underlying process distribution.

The synthetic control chart is a combination of a signed-rank chart due to Bakir (2004) and confirming run length chart due to Bourke (1991). Zhang Wu and Trevor Spedding (2000) introduced a synthetic control chart to detect small shifts in the process mean . A Nonparametric Shewhart-type signed rank control chart based on runs due to Chakraborti and Eryilmaz (2007) are more attractive to the practitioner than chart proposed by Bakir (2004) , as they offer smaller false alarm rates and larger in-control average run-length. A nonparametric Shewhart-type synthetic control chart by Pawar and Shirke (2010) provide a nonparametric chart based on the signed-rank statistics to monitor shifts in the process median. This chart performs better than the nonparametric signed-rank chart given by Bakir (2004) and Chakraborti and Eryilmaz (2007). Before the year 2001, almost all the research papers on control charts are based on average run length (ARL) criterion. Wu et al. (2001) introduced 'Synthetic control chart for increases in fraction non conforming' based on 'Average Time to Signal' (ATS) criterion. The research paper of this criterion might have been introduced for the first time. There are some research papers based on ATS criterion like a GR control charts to detecting shifts in the process mean due to Gadre and Rattihalli (2004) and it gives a significant reduction in out of control ATS as compared to those of the synthetic control chart and $\overline{X}$ chart. In case of GR chart, they found optimal choices of all the three parameters (n, K, L). So, we develop a non-parametric GR control chart and it gives a significant reduction in out of control ATS as compared to those of a nonparametric control chart due to Pawar and Shirke (2010).

Remainder of the paper is organized as follows. In the following section, the nonparametric Schewhart-type synthetic control chart is explained. Next section describes the

GR control chart for detecting shifts in the process mean. In Section-5 a nonparametric GR chart is explained after listing the basic notations needed to construct a chart. Next section describes the design and a procedure of obtaining design parameters of a nonparametric GR chart. In Section-7 steady state ATS performance of nonparametric GR chart is studied. It is illustrated that the proposed chart performs better than nonparametric Schewhart-type synthetic control chart. Concluding remarks are included in the last section.

3. A nonparametric Shewhart-type synthetic control chart

A Control Chart Based on the Signed-Rank statistic

Let (X_{t1},..X_{t2},…,X_{tn}) be a random sample (subgroup) of size $n \geq 1$ observed from a continuous process with median θ at sampling instances t=1, 2, … It is assumed that the underlying process distribution is continuous, symmetric and that the in-control process median is known or specified to be equal to θ_0. Further it is assumed that θ_0 is known and when $\theta \neq \theta_0$ the process is out of control. Bakir (2004) provided a nonparametric control chart based on the signed-rank statistic for the r^{th} subgroup (X_{t1},..X_{t2},…X_{tn}), the signed-rank statistic is defined as :

$$\psi_t = \sum_{j=1}^{n} \text{Sign}(X_{tj} - \theta_0) R_{tj}^{+} \quad t = 1,2,\ldots,\ldots \quad (1)$$

where sign(u) = -1 ,0,1 if u<0, =0, >0

$$R_{tj}^{+} = 1 + \sum_{i=1}^{n} I(|(X_{ti} - \theta_0)| < |(X_{tj} - \theta_0)|)$$

with ***I***(a<b)=1 if a< b and 0 otherwise

Equation (1) can be rewrite as $\psi_t = 2w_t^{+} - \frac{n(n+1)}{2}$ …. (2)

where w_t^{+} is the well-known wilcoxon Signed-Rank Statistic (the sum of the ranks of the absolute values of the deviations corresponding to the positive deviations).

Let UCL denote the upper control limit corresponding to a positive sided control chart. The chart gives an out of control at the first sampling instance t for which $\psi_t \geq$ UCL. The following section is briefly describe CRL chart.

The Conforming Run Length Chart :

The CRL chart was originally developed for attribute quality control by Bourke (1991). *CRL* is the number of inspected units between two consecutive non conforming units (including the ending non conforming unit). The *CRL* chart uses the *CRL* as the charting statistic. The idea behind the *CRL* chart is that the conforming run length will change when the fraction non conforming 'p' in the process changes. The *CRL* is shortened as *p* increases and lengthened as *p* decreases. The charting statistic (CRL) follows a waiting time distribution with parameter *p.* The mean value of *CRL* (i.e. the average number of inspected units in a CRL sample) is

$$\mu_{CRL} = \frac{1}{p} \qquad \text{...... (3)}$$

and the cumulative distribution function (cdf) of CRL is,

$$F_p(CRL) = 1- (1-p)^{CRL}; \; CRL=1, 2, \ldots \qquad \text{....... (4)}$$

If their only concern is the detection of a increase in *p*, the lower control limit (denoted L) is sufficient for the *CRL* chart. If α_{CRL} is the specified /desired Type I error of the CRL chart and p_0 is the in-control fraction non-conforming, L can be derived from the following equation:

$$\alpha_{CRL} = F_{po}(L) = 1 - (1 - P_0)^{L},$$

which gives

$$L = \frac{\text{In}(1-\alpha_{CRL})}{\text{In}(1-p_0)} \qquad (5)$$

If the sample CRL (i.e, the charting statistic) is smaller than or equal to L then it is very likely that the fraction non-conforming p has increased and therefore, an out–of-control signal will be given. ARL_{CRL} is the average number of CRL samples required to detect change in p. The ARL_{CRL} is given by

$$ARL_{CRL} = \frac{1}{1-(1-p)^L}$$

Let ANS_{CRL} be the average number of the inspected units required to signal a fraction non conforming shift. It is given by

$$ANS_{CRL} = \mu_{CRL} X\, ARL_{CRL}$$

$$= \frac{1}{p(1-(1-p)^L)} \quad (6)$$

A Nonparametric Synthetic Control Chart

The nonparametric synthetic control chart is a combination of the nonparametric signed-rank control chart based on ψ_t (called the ψ_t chart hereafter) and the CRL chart. Basically, the operation of the nonparametric synthetic control chart is similar to that of the synthetic control chart for monitoring the process mean, except that the subgroup mean is replaced by the signed-rank statistic ψ_t and the upper control limit is changed accordingly.

Operation

The operation of the nonparametric synthetic control chart is as follows.

1. Decide on the upper control limit of the ψ_t chart and the lower limit L of the CRL chart. Design of these control parameters will be described shortly.
2. At each inspection point "t" take a random sample of n observations and calculate ψ_t
3. If ψ_t < UCL then the sample is called a conforming sample. If so, control flow goes back to step(2). Otherwise, the sample is called a non conforming sample and move to the next step.

4. Check the number of samples between the current and the last non-conforming sample (including the current sample). This number is taken as the value of the plotting statistic (i.e. CRL) of the CRL chart in the synthetic chart.
5. If this CRL is larger than the lower control limit of the CRL chart, then the process is thought to be under control and the charting procedure is continued (i.e. go back to Step (2)). Otherwise, the process is declared to be out of control and control flow goes to the next step.
6. Take the necessary action to find and remove the assignable cause(s).

Design

The synthetic chart has two parameters namely, L and UCL. For given in-control ARL and sample size n, the parameters L and UCL are obtained as follows:

We note that the in-control ARL of the synthetic chart is denoted by ARLs(0),

where

$$\mathrm{ARL}s(0) = \frac{1}{p(0)(1-(1-p(0))^{L})} \tag{7}$$

and $p(\delta)=p_r(\psi_t \geq \mathrm{UCL}\ |\theta=\theta_0+\delta|)$

Performance of the Synthetic Signed-Rank Control Chart :

When there is a shift in the process median, the distribution of charting statistic is difficult to obtain. Pawar and Shirke (2010) used simulation to obtain the ARL values for various values of the shift in the process median. A simulation study based on 10000 runs is performed with n=5 and n=10 when the corresponding desired values are 32 and 380. The simulation study is carried out for three continuous distribution namely the normal, double exponential and Cauchy distribution. These three distributions are symmetric about the median, but have different tail behavior. Here Pawar and Shirke (2010) found optimum values of UCL, L and ARL for various shift in the process median.

4.Group Run Control chart for detecting shifts in the process mean :

To monitor the process mean, Shewhart's $\overline{X}$ charts are widely used in industries. Shewhart's $\overline{X}$ chart detects large shifts in the process mean effectively, its performance is 'poor' in detecting small or moderate shifts. To detect small shifts in the process mean, Wu and Spedding (2000) proposed a synthetic control chart by combining the Shewhart's $\overline{X}$ chart and CRL chart. This synthetic chart gives better performance than $\overline{X}$ and CRL chart. A synthetic control chart declares the process as out of control if group based CRL$\leq$ L.

Gadre and Rattihalli (2004) proposed a control chart called 'Group Runs' (GR) control chart by combining the Shewhart's $\overline{X}$ chart with an extended version of CRL chart. GR chart declares the process as out control, if the first value of CRL$\leq$ L or successive values of CRL $\leq$ L for the first time.

There are some notations used in the construction of GR chart.

1. μ_o : In-control value of the process mean.
2. σ : The process variability.
3. ATS(δ) : The average number of units required by GR chart to detect a shift in process mean from μ_o to $\mu_o \pm \delta\sigma$.
4. δ_1= Design shift in the mean
5. n= Group size
6. k : coefficient used in control limits of sub chart
7. L= Lower limit of GR Chart.
8. Y_r= the r^{th} value of CRL when the groups are treated as unit.

= In other words, it is the number of groups inspected between $(r-1)^{th}$ and r^{th} non-conformed group, including the r^{th} non-confirmed group.

9. τ= The minimum required value of ATS(0).

Implementation of GR(n,k,L) chart :

GR chart for detecting small shifts in the process mean is a combination of Shewhart's $\overline{X}$ chart and an extended version of confirming run length chart. Implementation of GR chart is described as follows.

1. Inspect n items produced in succession, which constitutes the respective group.

2. Declare the group a conformed or non-conformed using $\overline{X}$ chart.

3. A process is said to be out of Statistical control, if either $Y_1 \leq L$ or two successive Y_r's are less than or equal to L for the first time.

4. When the process goes out control, necessary corrective action should be taken to reset and to resume it. Once the process restarts, move to step 1.

Design of GR Chart :

In the synthetic control chart, for the same problem, Wu and Spedding computed optimum values of parameters (k, L) for given group size (n). In case of GR chart, optimum choices of the three parameters (n, k, L) are computed. In designing the GR chart, the model is

Minimize $ATS(\delta_1)$
Subject to the constraint
$ATS(0) \geq \tau$

Let P be the probability of the group being non conformed.

$$P(\delta) = 1 - P(L_{\bar{x}\backslash s} < \overline{X} < U_{\bar{x}\backslash s} \mid \overline{X} \sim N(\mu_0 + \delta\sigma, \frac{\sigma}{\sqrt{n}}))$$

Here Y_r (r = 1, 2, ...) are independently and identically distributed (i.i.d) waiting time random variable with mean 1/p. Therefore, if N is the number of defective groups observed before declaring the process has become out of control with mean as follows :

$$E(N) = \frac{1}{(1-(1-P(\delta))^L)^2} \quad \text{referred by Bouke (1991)}$$

If a process indicates an out-of-control situation for the first time when N^{th} non-conformed group is observed then

$$ATS(\delta_1) = \frac{n}{P(\delta_1)} \frac{1}{(1-(1-P(\delta_1))^L)^2} \tag{8}$$

Thus, the optimization problem can be written in terms of (n, k, L) as

Minimize $ATS(\delta_1) = \frac{n}{P(\delta_1)} \frac{1}{(1-(1-P(\delta_1))^L)^2}$

Subject to the constraint

$$\frac{n}{P(0)} \frac{1}{(1-(1-P(0))^L)^2} \geq \tau$$

5. A nonparametric GR control chart (N-GR control chart) :

Notations and Terms :

In nonparametric GR control chart, the chart gives an out of control signal at the first sampling instance t for which $\psi_t \geq$ UCL where UCL denote the upper control limit corresponding to a positive-sided control chart. A nonparametric GRchart determines status of the process by plotting the values of CRL. There are some important notations used in the construction of nonparametric GR chart.

Notations :

1. θ_0 : In-control value of the process median.
2. ATS(δ) : The average number of units required by GR nonparametric chart to detect a shift in process median from θ_0 to $\theta_0 \pm \delta$.
3. δ_1= Design shift in the median
4. ψ_t= Wilcoxon Signed Statistic
5. k=Upper limit of GR Chart
6. n= Group size
7. L= Lower limit of GR Chart.
8. Y_r= the r^{th} value of CRL when the groups are treated as unit.

   = In other words, it is the number of groups inspected between $(r-1)^{th}$ and r^{th} non-conformed group, including the r^{th} non-confirmed group.
9. τ= The minimum required value of ATS(0).

Implementation of N-GR (n,k,L) chart :

A nonparametric GR control chart for detecting shifts in median is combination of nonparametric signed-rank control chart based on ψ_t and an extended version of confirming run length chart. It has a single horizontal line drawn at Y=L. Implementation of nonparametric GR chart is described as follows :

1. Inspect n items produced in succession, which constitutes the respective group.
2. Declare the group a conformed or non-conformed using CRL chart.
3. A process is said to be out of Statistical control, if either $Y_1 \leq L$ or two successive Y_r's are less than or equal to L for the first time.

4. When the process goes out control, necessary corrective action should be taken to reset and to resume it. Once the process restarts, move to step 1.

The constants (n, k, L) that govern the implementation of the nonparametric GR chart are known as design parameters of the chart and their optimal values can be obtained as discussed in design of nonparametric GR chart.

6. Design of nonparametric GR Chart :

In nonparametric shewhart-type synthetic control chart, for the same problem, Pawar and Shirke (2010) computed optimum values of control parameters (k, L) for given group size. In case of nonparametric GR chart, we obtain optimal choices of all three parameters (n, k, L). In designing the nonparametric GR chart, the model is

Minimize $ATS(\delta_1)$
Subject to the constraint
$ATS(0) \geq \tau$

Let P is the probability of the group being non conformed.

$$P(\delta)=P_r(\psi_t \geq UCL \mid \theta=\theta_0+\delta)$$

Here Y_r (r=1,2,...) are independently and identically distributed (i.i.d) waiting time random variable with mean 1/p. Therefore, if N is the number of defective groups observed before declaring the process has become out of control then,

$$E(N) = \frac{1}{(1-(1-P(\delta))^L)^2} \quad \text{referred by Bourke (1991)}$$

If a process signals out-of-control situation for the first time when N^{th} non-conformed group is observed then

$$ATS(\delta_1) = \frac{n}{P(\delta_1)} \frac{1}{(1-(1-P(\delta_1))^L)^2} \qquad (9)$$

Thus, the optimization problem (1) can be written in terms of (n, L, UCL) as

Minimize $ATS(\delta_1) = \frac{n}{P(\delta_1)} \frac{1}{(1-(1-P(\delta_1))^L)^2}$

Subject to the constraint

$$\frac{n}{P(0)} \frac{1}{(1-(1-P(0))^L)^2} \geq \tau$$

Example 1. The following in some sense standard values $\delta_1 = 0.2$ and τ =160 are considered. For these input parameters, values of the design parameters (n, L, UCL) for the non parametric Shewhart type, synthetic chart and non parametric Group Run chart along with respective $ATS(\delta_1)$ are as follows :

For Normal distribution :

1. **Non-parametric Shewhart Quality Control Chart : n=75 UCL=38 ATS(δ_1) = 78**
2. **Synthetic Chart : n =20 L = 3 UCL = 40 ATS(δ_1) = 40**
3. **Non-parametric Group Run Chart : n =13 L = 3 UCL = 20 ATS(δ_1) = 34**

For Double Exponential distribution :

1. **Non-parametric Shewhart Quality Control Chart : n=76 UCL=38 ATS(δ_1) = 77**
2. **Synthetic Chart : n =13 L = 3 UCL = 26 ATS(δ_1) = 32**
3. **Group Run Non-parametric Chart : n =12 L = 3 UCL = 18 ATS(δ_1) = 26**

For Cauchy distribution :

1. **Non-parametric Shewhart Quality Control Chart : n=74 UCL=37 ATS(δ_1) = 73**
2. **Synthetic Chart : n =9 L = 3 UCL = 18 ATS(δ_1) = 19**
3. **Group Run Non-parametric Chart : n =8 L = 3 UCL = 12 ATS(δ_1) = 15**

This shows that not only ATS(δ_1) of nonparametric GR chart is less than ATS(δ_1) of synthetic chart and nonparametric Shewhart, but also the group size(n) of the nonparametric GR chart is not exceeding the group size of nonparametric Shewhart type synthetic chart and nonparametric Shewhart.

Example 2. The following in some sense standard values of δ_1 = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 and τ =160 are considered. For these input parameters, values of the design parameters for the nonparametric Shewhart chart, nonparametric Shewhart type synthetic chart and nonparametric GR chart for various shifts are as follows :

Table 1 : The Optimum Values of Design Parameter n, L, UCL and ATS(δ_1) of the Nonparametric Shewhart chart, Nonparametric Synthetic Chart and Nonparametric Group Run Chart for Normal distribution.

Shift (δ)	Nonparametric Shewhart Chart			Synthetic Chart				Nonparametric Group Run Chart			
	n	UCL	ATS1 Value	n	L	UCL (K)	ATS1 Value	n	L	UCL (K)	ATS1 Value
0.2	75	38	78	20	3	40	40	13	3	20	**34**
0.4	75	38	74	11	4	22	19	8	3	12	**16**
0.6	75	38	75	8	4	16	13	7	4	11	**10**
0.8	76	38	75	6	1	9	9	5	2	5	**7**
1.0	75	38	74	6	1	9	7	5	2	5	**5**
1.2	75	38	74	5	1	8	6	4	1	4	**4**

Table 2 : The Optimum Values of Design Parameter n, L, UCL and ATS(δ_1) of the Nonparametric Shewhart chart, Nonparametric Synthetic Chart and Nonparametric Group Run Chart for Double Exponential distribution

Shift (δ)	Nonparametric Shewhart Chart			Synthetic Chart				Group Run Chart			
	n	UCL	ATS1 Value	n	L	UCL	ATS1 Value	n	L	UCL	ATS1 Value
0.2	76	38	77	13	3	26	32	12	3	18	**26**
0.4	75	38	74	9	3	18	16	5	4	8	**12**
0.6	75	38	74	5	7	10	11	4	4	6	**7**
0.8	75	38	74	5	6	10	8	5	2	5	**6**
1.0	75	38	74	5	1	8	7	4	3	6	**5**
1.2	75	38	74	4	1	6	5	4	1	4	**4**

Table 3 : The Optimum Values of Design Parameter n, L, UCL and ATS(δ_1) of the Nonparametric Shewhart chart, Nonparametric Synthetic Chart and Nonparametric Group Run Chart for Cauchy distribution.

Shift (δ)	Nonparametric Shewhart Chart			Synthetic Chart				Group Run Chart			
	n	UCL	ATS1 Value	n	L	UCL	ATS1 Value	N	L	UCL	ATS1 Value
0.2	74	37	73	8	3	18	19	8	3	12	**15**
0.4	76	38	75	5	7	10	9	4	3	6	**7**
0.6	75	38	74	5	8	10	7	4	3	6	**5**
0.8	74	37	73	4	1	6	6	4	4	6	**4**
1.0	74	37	73	4	1	6	5	4	3	6	**4**
1.2	73	37	72	4	1	6	5	4	3	6	**4**

For further comparison of nonparametric GR chart with Nonparametric Shewhart Chart and nonparametric shewhart-type synthetic control chart for various values of δ the normalized ATS (normalized with respect to Nonparametric Shewhart chart) values are computed. For nonparametric shewhart chart the normalized ATS is always unity. The values are given in Table 4, Table 5 and Table 6 for Normal distribution, Double exponential and Cauchy distribution respectively. Entries from δ = 0 to δ=1.35 are listed. For the larger values of δ, the values of normalized ATS remains the same from δ=1.35.

Table 4 : Performance of the N-GR Chart , Nonparametric Shewhart's Synthetic Chart and Nonparametric Shewhart Chart for various values of delta for Normal Distribution.

Δ	ATS Shewhart	ATS Synthetic	ATS N-GR	Normalized ATS Shewhart	Normalized ATS Synthetic	Normalized ATS N-GR
0.01	153.1863	136.9546	153.0125	1.1185	1	1.1172
0.02	142.6398	127.4737	128.5199	1.1190	1	1.0082
0.03	133.6422	110.3425	114.6462	1.2112	1	1.0390
0.04	124.6468	105.3843	102.2186	1.1828	1	0.9700
0.05	119.2369	94.7577	96.9570	1.2583	1	1.0232
0.1	96.9869	64.9321	61.9959	1.4937	1	0.9548
0.15	84.7937	48.4692	43.1985	1.7494	1	0.8913
0.2	79.3315	39.1156	32.6353	2.0281	1	0.8343
0.25	76.6323	32.6572	26.5170	2.3466	1	0.8120
0.3	75.5059	28.1721	22.5860	2.6802	1	0.8017
0.35	75.1428	25.6249	19.4814	2.9324	1	0.7603
0.4	75.0525	23.7036	17.6176	3.1663	1	0.7432
0.45	75	22.3594	16.2860	3.3543	1	0.7284
0.5	75	21.4660	15.3838	3.4939	1	0.7167
0.55	75	21.0218	14.6317	3.5677	1	0.6960
0.6	75	20.6854	14.1817	3.6257	1	0.6856
0.65	75	20.3150	13.7984	3.6919	1	0.6792
0.7	75	20.2061	13.5791	3.7117	1	0.6720
0.75	75	20.1309	13.3461	3.7256	1	0.6630
0.8	75	20.0622	13.2737	3.7384	1	0.6616
0.85	75	20.0321	13.1367	3.7440	1	0.6558
0.9	75	20.0200	13.1062	3.7463	1	0.6547
0.95	75	20.0080	13.0666	3.7485	1	0.6531
1	75	20.0000	13.0221	3.7500	1	0.6511

1.05	75	20.0020	13.0143	3.7496	1	0.6507
1.1	75	20.0000	13.0091	3.7500	1	0.6505
1.15	75	20.0000	13.0052	3.7500	1	0.6503
1.2	75	20.0000	13.0091	3.7500	1	0.6505
1.25	75	20.0000	13.0026	3.7500	1	0.6501
1.3	75	20.0000	13.0000	3.7500	1	0.6500
1.35	75	20.0000	13.0000	3.7500	1	0.6500

Figure 1 : The ATS-curves of the Three chart-Types as function of for various values of delta for Normal Distribution.

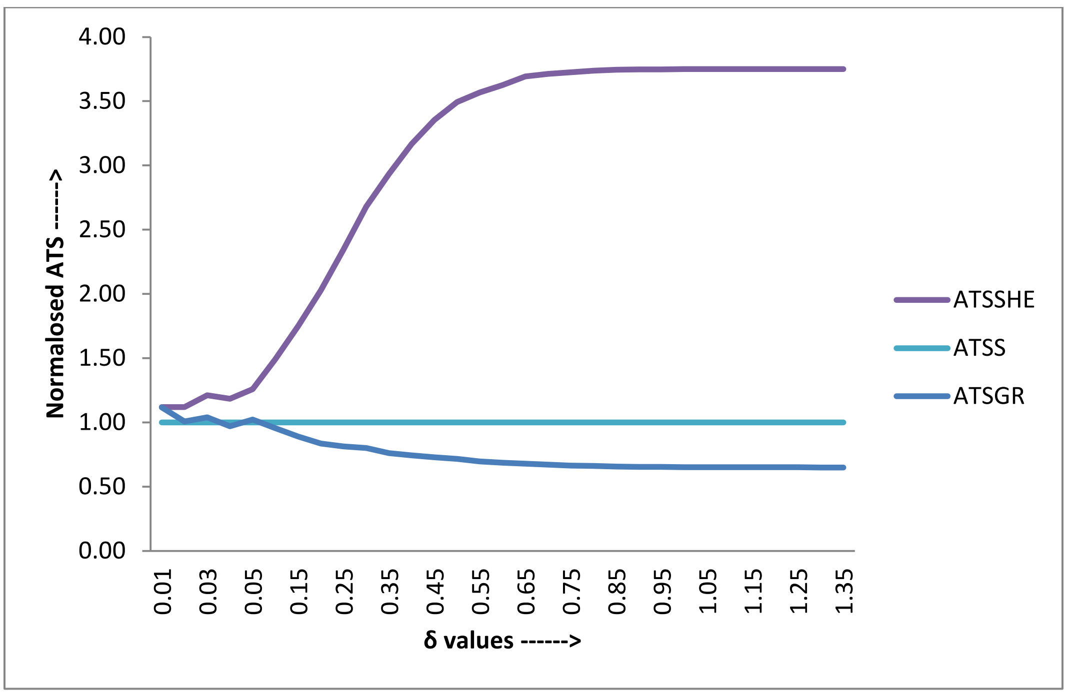


Table 5 : Performance of the N-GR Chart , Nonparametric Shewhart's Synthetic Chart Nonparametric Shewhart Chart and for various values of delta for Double Exponential Distribution.

delta	ATS Shewhart	ATS Synthetic	ATS N-GR	Normalized ATS Shewhart	Normalized ATS Synthetic	Normalized ATS N-GR
0.01	149.6947	124.5225	115.7261	1.2021	1	0.9294
0.02	137.9561	114.7505	98.5906	1.2022	1	0.8592
0.03	129.3397	106.9216	88.0106	1.2097	1	0.8231
0.04	121.6584	96.9715	80.5942	1.2546	1	0.8311
0.05	114.7863	86.2218	71.8344	1.3313	1	0.8331
0.1	91.0071	56.3106	43.9304	1.6162	1	0.7801
0.15	81.9407	39.7681	32.3546	2.0605	1	0.8136
0.2	77.6144	32.1766	24.4275	2.4121	1	0.7592
0.25	76.5049	24.6031	19.7841	3.1096	1	0.8041
0.3	76.0913	20.8518	17.5085	3.6491	1	0.8397
0.35	76.0304	18.5425	15.8225	4.1003	1	0.8533
0.4	76	16.9041	14.7729	4.4960	1	0.8739
0.45	76	15.8570	13.9290	4.7928	1	0.8784
0.5	76	15.0788	13.3867	5.0402	1	0.8878
0.55	76	14.4408	13.0144	5.2629	1	0.9012
0.6	76	14.1096	12.6835	5.3864	1	0.8989
0.65	76	13.8076	12.5003	5.5042	1	0.9053
0.7	76	13.4903	12.3821	5.6337	1	0.9178
0.75	76	13.3831	12.2501	5.6788	1	0.9153
0.8	76	13.2654	12.1853	5.7292	1	0.9186
0.85	76	13.2061	12.1200	5.7549	1	0.9178
0.9	76	13.1075	12.0944	5.7982	1	0.9227
0.95	76	13.0851	12.0555	5.8082	1	0.9213
1	76	13.0653	12.0422	5.8169	1	0.9217
1.05	76	13.0496	12.0144	5.8239	1	0.9207
1.1	76	13.0391	12.0216	5.8286	1	0.9220
1.15	76	13.0156	12.0084	5.8391	1	0.9226
1.2	76	13.0078	12.0108	5.8426	1	0.9234
1.25	76	13.0039	12.0012	5.8444	1	0.9229
1.3	76	13.0104	12.0024	5.8415	1	0.9225
1.35	76	13.0039	12.0036	5.8444	1	0.9231

Figure 2 : The ATS-curves of the Three chart-Types as function of for various values of delta for Double Exponential Distribution.

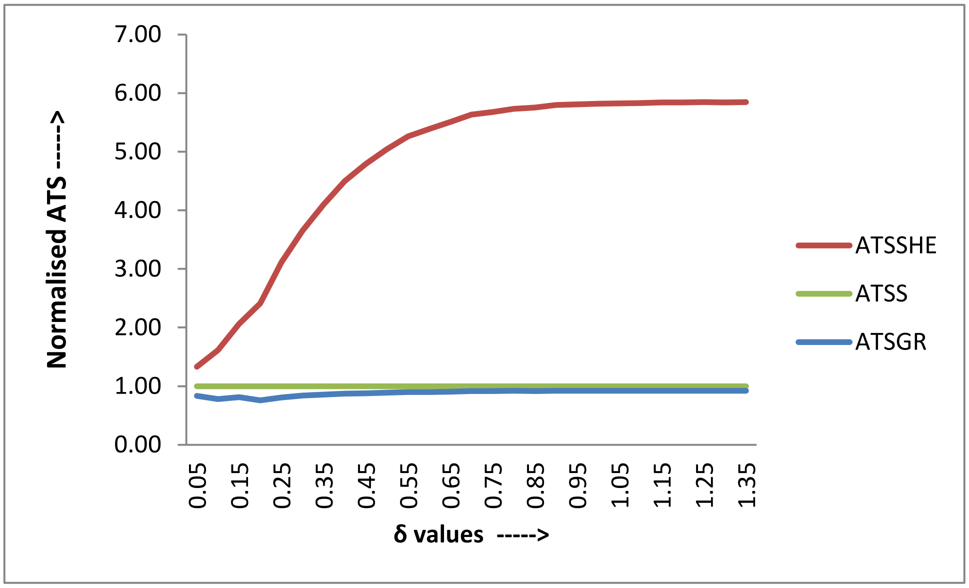
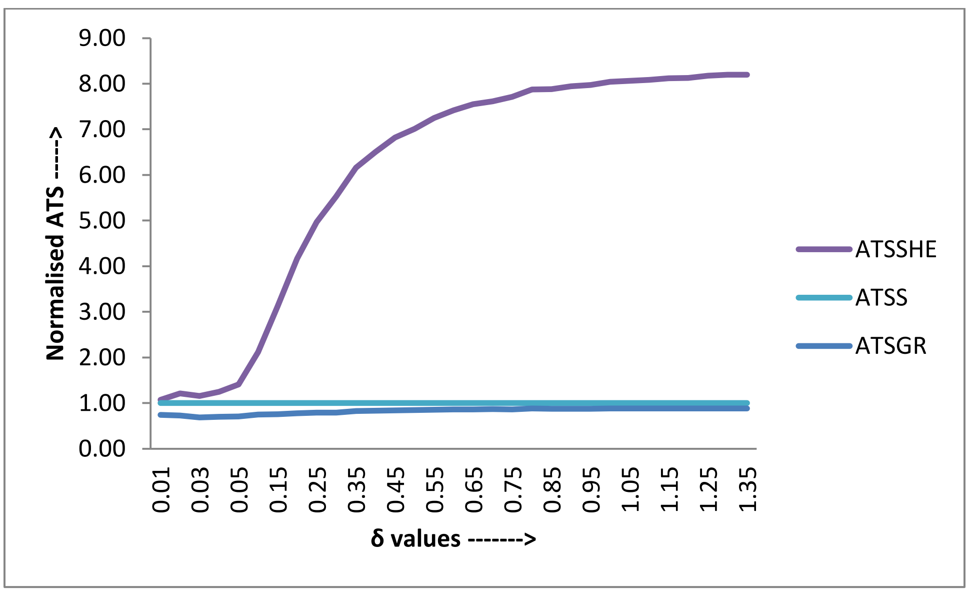


Table 6 : Performance of the N-GR Chart, Nonparametric Shewhart's Synthetic Chart Nonparametric Shewhart Chart and for various values of delta for Cauchy Distribution

delta	ATS Shewhart	ATS Synthetic	ATS N-GR	Normalized ATS Shewhart	Normalized ATS Synthetic	Normalized ATS N-GR
0.01	141.8704	132.4487	98.4358	1.071135	1	0.7432
0.02	126.0992	103.8314	75.8764	1.214461	1	0.7308
0.03	112.4094	97.3812	66.7482	1.154324	1	0.6854
0.04	102.7721	82.4368	57.8810	1.246678	1	0.7021
0.05	95.89905	68.0730	48.1490	1.408767	1	0.7073
0.1	79.53118	37.5635	28.0626	2.117244	1	0.7471
0.15	76.48184	24.4857	18.5384	3.123531	1	0.7571
0.2	76.11417	18.2495	14.2223	4.170763	1	0.7793
0.25	76.0076	15.3050	12.1489	4.966184	1	0.7938
0.3	76	13.7378	10.9110	5.532197	1	0.7942
0.35	76	12.3375	10.1909	6.160081	1	0.8260
0.4	76	11.6877	9.7086	6.502557	1	0.8307
0.45	76	11.1370	9.3464	6.824081	1	0.8392
0.5	76	10.8451	9.1764	7.007754	1	0.8461
0.55	76	10.4850	8.9213	7.248485	1	0.8509
0.6	76	10.2510	8.8010	7.413896	1	0.8585

0.65	76	10.0675	8.6938	7.549029	1	0.8635
0.7	76	9.9781	8.6319	7.616643	1	0.8651
0.75	76	9.8542	8.5051	7.712452	1	0.8631
0.8	76	9.6513	8.4922	7.874588	1	0.8799
0.85	76	9.6461	8.4232	7.878861	1	0.8732
0.9	76	9.5683	8.3990	7.942885	1	0.8778
0.95	76	9.5346	8.3239	7.971007	1	0.8730
1	76	9.4528	8.3169	8.039922	1	0.8798
1.05	76	9.4259	8.2874	8.062861	1	0.8792
1.1	76	9.4012	8.2814	8.084086	1	0.8809
1.15	76	9.3580	8.2436	8.121414	1	0.8809
1.2	76	9.3541	8.2427	8.124806	1	0.8812
1.25	76	9.2978	8.2172	8.173955	1	0.8838
1.3	76	9.2748	8.2062	8.194276	1	0.8848
1.35	76	9.2748	8.1801	8.194276	1	0.8820

Figure 3 : The ATS-curves of the Three chart-Types as function of for various values of delta for Cauchy Distribution.



The graph of normalised ATS against values related to the data in Table 4, 5 and 6 are given in Figure 1, 2 and 3 respectively, from which it is observed that for $\delta > 0$ we have $ATS_g(\delta_1) < ATS_s(\delta_1)$ and $ATS_g(\delta_1) < ATS_{sh}(\delta_1)$. Thus, the nonparametric GR chart detects a shift of any size in the process median earlier than the nonparametric Shewhart chart and non parametric Shewhart-type synthetic control chart, though optimum values of the design parameters are computed for a specific δ value for these three distribution.

7. Runs Rules Representation and Steady State ATS Performance of N-GR Chart.

Davis and Woodall have discussed the runs rules representation of the synthetic control chart identifying shifts in the process mean depending on the X-bar procedure. Here we discuss the runs rule representation of a N-GR chart, using the group run lengths. Let the values of CRL be classified as '0' and '1' according as CRL>L and CRL ≤ L. Thus, the sequence of CRLs can be represented as a string of zeros and ones. "Declare the process as out of control if two successive Y_rs are less than or equal to L.

Thus, N-GR chart is identical to the above stated runs rule with the head start (Yo ≤ L). The formula for ATS can easily obtained by using transition probability matrix (t.p.m) of an absorbing Markov chain based on CRL values used to model the N-GR chart. If m={Y>L} and *l* ={Y≤ L}, then following t.p.m. is obtained related to the N-GR chart.

	m	*l*	Signal
m	1-A	A	0
l	1-A	0	A
Signal	0	0	1

According to the runs rule representation of N-GR chart, it is clear that *l* corresponds to the initial state. Let R be the matrix obtained by deleting the last row and column of the above matrix. Then, the average number of defective groups observed before declaring the process as out of control is identical to the average time for the Markov chain to enter the absorbing state. The vector of average times corresponding to the various initial states is given by

$$E(N) = (1\text{-}R)^{-1}\,\underline{1}$$

Where 1 is a column vector of order two having all elements unity. The second element of E(N) is $\frac{1}{A^2}$.

Let the groups be classified as '0' or '1' according as conformed or non conformed and for illustration purpose assume that L=2. Thus the GR chart will produce a signal if Y_1 or two successive Y_r^s are less than or equal to 2. Let Q and P be respectively the probabilities of the group being conformed and non conformed. As in Davis and Woodall a Markov chain representation in this situation can be described by using the 8 states listed and the t.p.m. given below. Permitted

Table 7 : The States of the Markov Chain and their Labels for L=2

1	00	5	00101
2	001	6	00110
3	0010	7	001010
4	0011	8	Signal

In the above table, state 1 indicates that by now more than or equal to two (which is the value of L) conforming groups are observed, while state 2 indicates that a non confirming group being observed after a sequence of at least two conforming groups. State 8 (signal) is a absorbing state. It indicates sequences of 0's and 1's ending with 1 and the current and the previous run length not exceeding L. The other states are non-absorbing states, which are accessible from state 2. The related one step t.p.m. is given below.

	1	2	3	4	5	6	7	8
1	Q	P	0	0	0	0	0	0
2	0	0	Q	P	0	0	0	0
3	Q	0	0	0	P	0	0	0
4	0	0	0	0	0	Q	0	P
5	0	0	0	0	0	0	Q	P
6	Q	0	0	0	0	0	0	P
7	Q	0	0	0	0	0	0	P
8	0	0	0	0	0	0	0	1

Consider another illustration where assume that L=3. Markov chain representation in this situation can be described by using the 14 states listed and the t.p.m. given below :

Table 7.1 : The States of the Markov Chain and their Labels L=3

1	000	8	000110
2	0001	9	0001010
3	00010	10	00010010
4	000100	11	0001100
5	00011	12	00010100
6	000101	13	000100100
7	0001001	14	Signal

The related one step t.p.m. is given below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Q	P	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	Q	0	P	0	0	0	0	0	0	0	0	0
3	0	0	0	Q	0	P	0	0	0	0	0	0	0	0
4	Q	0	0	0	0	0	P	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	Q	0	0	0	0	0	P
6	0	0	0	0	0	0	0	0	Q	0	0	0	0	P
7	0	0	0	0	0	0	0	0	0	Q	0	0	0	P
8	0	0	0	0	0	0	0	0	0	0	Q	0	0	P
9	0	0	0	0	0	0	0	0	0	0	0	Q	0	P
10	0	0	0	0	0	0	0	0	0	0	0	0	Q	P
11	Q	0	0	0	0	0	0	0	0	0	0	0	0	P
12	Q	0	0	0	0	0	0	0	0	0	0	0	0	P
13	Q	0	0	0	0	0	0	0	0	0	0	0	0	P
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Consider another illustration where assume that L=4. Markov chain representation in this situation can be described by using the 22 states listed and the t.p.m. given below

Table 7.2 : The States of the Markov Chain and their Labels L=4

1	0000	12	000010010
2	00001	13	0000100010
3	000010	14	00001100
4	0000100	15	000010100
5	00001000	16	0000100100
6	000011	17	00001000100
7	0000101	18	000011000
8	00001001	19	0000101000

9	000010001	20	00001001000
10	0000110	21	000010001000
11	00001010	22	Signal

The related one step t.p.m. is given below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Sig
1	Q	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	Q	0	0	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	Q	0	0	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	Q	0	0	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Q	0	0	0	0	0	0	0	P	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	0	0	0	0	0	P
7	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	0	0	0	0	P
8	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	0	0	0	P
9	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	0	0	P
10	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	0	P
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	0	P
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	0	P
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	0	P
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	0	P
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	0	P
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	0	P
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Q	P
18	Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	P
19	Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	P
20	Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	P
21	Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	P
Sig	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

For the general value of L, the matrix R_1, (eliminating the last row and the last column of the t.p.m.) has the following states.

1. A sequence of L zeros; State 1 in the above Table 7.1.
2. Sequence of L zeros followed by 1 and further appended by at most (L-1) zeros. There are L such sequences; State 2,3 and 4 in the above example. Thus, such type of states are 3 (=L).

3. Each of the sequences in (2) followed by 1 and is further appended by a sequence of at most (L-1) zeros. The total number of such sequences is L^2; States 5 to 13 in the above example. Thus, such type of states are 9 ($=L^2$).

Therefore, R_1 is a square matrix of order $1 + L + L^2 = L(L+1) + 1$

Note that the (i, j)th element of R_1 is

$$R_1(i, j) = \begin{cases} C & \text{if the ith state leads to the jth state, which corresponds to a sequence ending with `0'} \\ D & \text{if the ith state leads to the jth state, which corresponds to a sequence ending with `1'} \\ 0 & \text{otherwise} \end{cases}$$

Let $\underline{\pi}$ be a 1 X (L(L+1)+1) row vector corresponding to the stationary probability distribution that the Markov chain will be in each of the non absorbing states, which is conditioned on no signal. Then the steady state ATS of the N-GR chart can be obtained after multiplying by n_{ng} to the product of $\underline{\pi}$ and ARL, where ARL can be obtained by $(I- R_1)^{-1}\,\underline{1}$.

It is to be noted that for any run length based control chart, the steady state ATS is not smaller than zero state ATS. If the signal depends on one point only, both ATS' are same. Hence the performance of the two charts should be compared by making the $(S.S.\ ATS)_o$ of the two charts same. Hence we compute adjusted steady state ATS of the Chart II with respect to the chart I as

$$[Adj.S.S.ATS(\delta)]II = \frac{[S.S.\ ATS(\delta)]II}{[S.S.ATS(0)]II} \text{ x } [S.S.\ ATS(0)]I$$

Example 1 (Cont.)

The following table gives the steady ATS and the adjusted steady state ATS values corresponding to various values of δ of Example 1 for all three charts.

Table 8 : Values of Steady State (ATS) corresponding to various δ Values for the three charts considered in Example 1 (Normal Distribution)

Δ	S.S.ATS Shewhart	S.S.ATS Synthetic	Adj|S.S.ATS|s	S.S.ATS N- Gr	Adj|S.S.ATS |N- Gr
0	163.0426	186.44	163	217.0606	163
0.1	97.4026	87.9196	76.88607689	94.7769	71.19059007
0.2	78.9474	52.4896	45.90238606	53.5918	40.25487081
0.3	75.9474	37.5787	32.86273843	36.0265	27.06089557
0.4	75	30.9202	27.03985089	26.7773	20.11346423
0.5	75	27.2882	23.86365092	22.6678	17.02666006
0.6	75	25.9328	22.67834766	20.232	15.1970366
0.7	75	25.3035	22.12802204	19.0544	14.31249576

Table 9 : Values of Steady State (ATS) corresponding to various δ Values for the three charts considered in Example 1 (Double Exponential Distribution)

Δ	S.S.ATS Shewhart	S.S.ATS Synthetic	Adj|S.S.ATS|s	S.S.ATS N- Gr	Adj|S.S.ATS |N- Gr
0	165.2174	177.8496	165	183.6527	165
0.1	90.4762	73.474	68.25533061	68.3245	61.46599667
0.2	78.3505	41.081	38.16312215	39.3253	35.37777457
0.3	76	28.4956	26.47163077	27.4547	24.69876104
0.4	76	22.7919	21.17304992	22.0563	19.84225955
0.5	76	19.2446	17.87770552	19.2688	17.33457247
0.6	76	17.972	16.69549503	18.1167	16.29812178
0.7	76	17.0684	15.85607542	17.3355	15.59533967
0.8	76	16.6493	15.46674301	16.8492	15.1578551
0.9	76	16.4473	15.27909055	16.8492	15.1578551

Table 10 : Values of Steady State (ATS) corresponding to various δ Values for the three charts considered in Example 1 (Cauchy Distribution)

Δ	S.S.ATS Shewhart	S.S.ATS Synthetic	Adj|S.S.ATS|s	S.S.ATS N- Gr	Adj|S.S.ATS |N- Gr
0	160.8696	183.8727	161	161.2166	161
0.1	77.8947	50.8666	50.82540121	41.708	41.61822838
0.2	74	24.832	24.81188762	22.1702	22.12248122
0.3	74	18.2133	18.19854836	16.8148	16.7786081
0.4	74	15.2677	15.25533411	14.444	14.41291097
0.5	74	14.1228	14.11136141	13.2663	13.23774583
0.6	74	13.1372	13.12655968	12.8459	12.81825069
0.7	74	12.609	12.59878749	12.2614	12.23500876
0.8	74	12.2797	12.26975421	11.8993	11.87368814
0.9	74	12.1214	12.11158242	11.7258	11.70056158
1	74	11.9671	11.95740739	11.7258	11.70056158
1.1	74	11.8166	11.80702929	11.557	11.5321249
1.2	74	11.8166	11.80702929	11.557	11.5321249

From the above table, we conclude that, the steady state performance of N-GR is better than that of the two other charts.

Conclusions

A N-GR control chart proposed here performs significantly better than Nonparametric Shewhart Chart and Nonparametric Shewhart's Synthetic Chart for all three distributions. The ATS(δ_1) is significantly less than for the other two charts for all three distributions. Also in steady state N-GR performs better than the remaining two charts.

In following section we are concentrate on ARL based comparison between a nonparametric GR control chart and a nonparametric Shewhart-type synthetic control chart.

Design :

The synthetic chart has two parameters namely, L and UCL. For given in control ARL and subgroup sample size n, the parameters L and UCL are obtained as follows:

Suppose the desired in-control ARL is ARLo and the subgroup sample size is n. We computer the ARLs(0) values using equation (7) for UCL= 1, 2, …, (n(n+1)/2) and L=1, 2, … and choose that pair of (L, UCL) for which the ARLs(0) is close to ARLo.

Table 11 : In control ARL values for positive sided chart for various values of UCL and L for a nonparametric Shewhart-type synthetic control chart (For n=5)

	<---- L ----->									
UCL	1	2	3	4	5	6	7	8	9	10
1	4.00	2.67	2.29	2.13	2.06	2.03	2.02	2.01	2.00	2.00
2	6.06	3.80	3.11	2.81	2.66	2.57	2.53	2.50	2.48	2.47
3	6.06	3.80	3.11	2.81	2.66	2.57	2.53	2.50	2.48	2.47
4	10.24	6.07	4.74	4.12	3.78	3.58	3.45	3.37	3.31	3.28
5	10.24	6.07	4.74	4.12	3.78	3.58	3.45	3.37	3.31	3.28
6	20.89	11.73	8.73	7.28	6.45	5.91	5.56	5.31	5.13	4.99
7	20.89	11.73	8.73	7.28	6.45	5.91	5.56	5.31	5.13	4.99
8	40.93	22.22	16.02	12.97	11.18	10.01	9.20	8.61	8.17	7.83
9	40.93	22.22	16.02	12.97	11.18	10.01	9.20	8.61	8.17	7.83
10	113.66	59.69	41.67	**32.74**	27.41	23.89	21.42	19.55	18.13	17.02
11	113.66	59.69	41.67	**32.74**	27.41	23.89	21.42	19.55	18.13	17.02
12	256.00	132.13	90.90	70.32	58.01	49.83	44.02	39.67	36.32	33.65
13	256.00	132.13	90.90	70.32	58.01	49.83	44.02	39.67	36.32	33.65
14	1024.00	520.13	352.23	268.32	218.01	184.49	160.58	142.67	128.75	117.64
15	1024.00	520.13	352.23	268.32	218.01	184.49	160.58	142.67	128.75	117.64

Table 12 : In control ARL values for positive sided chart for various values of UCL and L for a Nonparametric Group Run Control Chart (For n=5)

	<---- L ----->									
UCL	1	2	3	4	5	6	7	8	9	10
1	8	3.56	2.61	2.28	2.13	2.06	2.03	2.02	2.01	2
2	14.91	5.87	3.94	3.21	2.87	2.69	2.59	2.54	2.51	2.49
3	14.91	5.87	3.94	3.21	2.87	2.69	2.59	2.54	2.51	2.49
4	**32.77**	11.51	7.02	5.31	4.47	4	3.72	3.55	3.43	3.36
5	**32.77**	11.51	7.02	5.31	4.47	4	3.72	3.55	3.43	3.36
6	95.47	**30.09**	16.69	11.6	9.09	7.65	6.76	6.16	5.75	5.45
7	95.47	**30.09**	16.69	11.6	9.09	7.65	6.76	6.16	5.75	5.45
8	261.89	77.04	**40.1**	26.29	19.52	15.65	13.22	11.58	10.43	9.58
9	261.89	77.04	**40.1**	26.29	19.52	15.65	13.22	11.58	10.43	9.58
10	1211.69	333.47	162.89	100.54	70.49	**53.54**	**42.96**	**35.86**	**30.85**	27.16
11	1211.69	333.47	162.89	100.54	70.49	**53.54**	**42.96**	**35.86**	**30.85**	27.16
12	4096	1091.13	516.38	309.08	210.34	155.21	121.09	98.38	82.43	70.75
13	4096	1091.13	516.38	309.08	210.34	155.21	121.09	98.38	82.43	70.75
14	32768	8454.13	3876.97	2249.82	1485.21	1063.68	805.81	636.05	518.03	432.45
15	32768	8454.13	3876.97	2249.82	1485.21	1063.68	805.81	636.05	518.03	432.45

From Table 12 we choose the pairs of (L,UCL) for which the ARLs(0) is close to ARL0=32. The following table shows ARLs calculations for various combination of (L, UCL) and shift median for three distributions.

Table 13 : ARL values for positive sided charts for Normal Distribution (n=5)

UCL=4, L=1			UCL=5, L=1		UCL=6, L=2		UCL=7, L=2		UCL=8, L=3	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	32.77	33.28	32.77	31.66	30.09	30.3	30.09	30.33	40.01	40.96
0.2		9.46		9.47		3.35		3.33		3.54
0.4		4.05		3.85		1.79		1.78		1.84
0.6		2.12		2.09		1.44		1.44		1.46
0.8		1.46		1.46		1.31		1.31		1.31
1		1.19		1.18		1.23		1.23		1.24
1.2		1.07		1.07		1.19		1.19		1.19

UCL=9, L=3			UCL=10, L=6		UCL=11, L=6		UCL=10, L=7		UCL=11, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	40.01	40.01	53.54	53.54	53.54	52.88	42.96	41.43	42.96	43.01
0.2		3.58		3.6		3.56		3.39		3.33
0.4		1.83		1.86		1.87		1.85		1.86
0.6		1.46		1.5		1.51		1.5		1.5
0.8		1.31		1.35		1.35		1.35		1.35
1		1.24		1.26		1.27		1.26		1.27
1.2		1.2		1.21		1.21		1.21		1.21

UCL=10, L=8			UCL=11, L=8		UCL=10, L=9		UCL=11, L=9	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	35.86	35.68	35.86	34.36	30.85	117.82	30.85	117.82
0.2		3.22		8.77		19.31		19.31
0.4		1.84		3.76		6.01		6.01
0.6		1.5		2.36		3.1		3.1
0.8		1.35		1.69		2.1		2.1
1		1.27		1.37		1.58		1.58
1.2		1.21		1.2		1.33		1.33

Table 14 : ARL values for positive sided charts for Cauchy Distribution (n=5)

UCL=4, L=1			UCL=5, L=1		UCL=6, L=2		UCL=7, L=2		UCL=8, L=3	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	32.77	33.16	32.77	33.12	30.09	29.81	30.09	30.32	40.01	40.94
0.2		3.45		3.46		3.36		3.32		3.6
0.4		1.71		1.7		1.79		1.78		1.82
0.6		1.33		1.33		1.44		1.45		1.46
0.8		1.19		1.19		1.31		1.3		1.31
1		1.13		1.13		1.23		1.23		1.24
1.2		1.09		1.09		1.19		1.19		1.2

UCL=9, L=3			UCL=10, L=6		UCL=11, L=6		UCL=10, L=7		UCL=11, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	40.01	39.05	53.54	53.72	53.54	53.94	42.96	41.04	42.96	40.72
0.2		3.58		3.52		3.57		3.35		3.34
0.4		1.84		1.88		1.87		1.85		1.84
0.6		1.46		1.5		1.5		1.5		1.5
0.8		1.32		1.35		1.35		1.34		1.35
1		1.24		1.27		1.26		1.26		1.26
1.2		1.19		1.21		1.21		1.21		1.21

UCL=10, L=8	UCL=11, L=8	UCL=10, L=9	UCL=11, L=9

Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	35.86	36.56	35.86	34.36	30.85	117.82	30.85	117.82
0.2		3.25		8.77		19.31		19.31
0.4		1.84		3.76		6.01		6.01
0.6		1.498		2.36		3.1		3.1
0.8		1.35		1.69		2.1		2.1
1		1.26		1.37		1.58		1.58
1.2		1.21		1.2		1.33		1.33

Table 15 :ARL values for positive sided charts for Double Exponential Distribution (n=5)

	UCL=4, L=1		UCL=5, L=1		UCL=6, L=2		UCL=7, L=2		UCL=8, L=3	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	32.77	32.98	32.77	32.62	30.09	30.83	30.09	29.94	40.01	39.51
0.2		6.77		6.85		5.92		5.91		7.03
0.4		2.78		2.79		2.49		2.46		2.74
0.6		1.7		1.7		1.58		1.57		1.69
0.8		1.31		1.31		1.26		1.26		1.3
1		1.14		1.14		1.12		1.13		1.18
1.2		1.06		1.06		1.06		1.06		1.1

	UCL=9, L=3		UCL=10, L=6		UCL=11, L=6		UCL=10, L=7		UCL=11, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	40.01	40.72	53.54	51.91	53.54	54.01	42.96	39.86	42.96	43.54
0.2		7.09		7.8		7.74		6.85		6.82
0.4		2.73		3		3.02		2.86		2.87
0.6		1.69		1.91		1.91		1.89		1.9
0.8		1.32		1.49		1.49		1.49		1.49
1		1.18		1.28		1.28		1.28		1.28
1.2		1.09		1.17		1.17		1.17		1.17

	UCL=10, L=8		UCL=11, L=8		UCL=10, L=9		UCL=11, L=9	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	35.86	35.44	35.86	34.36	30.85	117.82	30.85	117.82
0.2		6.11		8.77		19.31		19.31
0.4		2.76		3.76		6.01		6.01
0.6		1.87		2.36		3.1		3.1
0.8		1.5		1.69		2.1		2.1
1		1.29		1.37		1.58		1.58
1.2		1.17		1.2		1.33		1.33

F

rom the above tables the results are shown in Tables 16, 17 and 18 for subgroup of size n=5 under the normal, the double exponential, and the Cauchy distribution. Due to Bakir (2004), the scale parameter is set to be $\lambda=1/\sqrt{2}$ for the double exponential distribution to achieve a standard deviation of 1.0. For the Cauchy distribution, $\lambda=0.2605$ is chosen a tail probability of 0.05 above $\theta+1.645$, the same as that of a normal distribution with a mean θ and a standard deviation of 1.0. For each chart, the control limits are found such that the in-control ARL is 32 for subgroup of size n=5.

Table 16 : ARL values for positive sided charts for N-Group Runs Chart
Normal distribution (n=5)

Shift ($\theta-\theta_o$)	N-Group Runs UCL=10 L=8	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	35.68	32.77	25.47	32.00	32.00
0.2	3.22	10.06	10.49	15.60	12.74
0.4	1.84	4.51	5.45	8.22	6.01
0.6	1.50	2.49	3.60	5.00	3.32
0.8	1.35	1.75	2.74	3.31	2.12
1.0	1.27	1.38	2.34	2.40	1.55
1.2	1.19	1.19	2.14	1.84	1.26

Table 17 : ARL values for positive sided charts for N-Group Runs Chart
Normal distribution (n=5)

Shift ($\theta-\theta_o$)	N-Group Runs UCL=4 L=1	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	33.28	32.77	25.47	32.00	32.00
0.2	9.46	10.06	10.49	15.60	12.74
0.4	4.05	4.51	5.45	8.22	6.01
0.6	2.12	2.49	3.60	5.00	3.32
0.8	1.46	1.75	2.74	3.31	2.12
1.0	1.19	1.38	2.34	2.40	1.55
1.2	1.07	1.19	2.14	1.84	1.26

Table 18 : ARL values for positive sided charts for N-Group Runs Chart double exponential distribution (n=5)

Shift $(\theta - \theta_o)$	N-Group Runs UCL=10 L=8	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	35.44	32.77	25.47	32.00	32.00
0.2	6.11	7.45	8.58	10.40	13.83
0.4	2.76	3.17	5.46	5.31	6.58
0.6	1.87	1.96	3.20	3.30	3.51
0.8	1.50	1.52	2.62	2.38	2.16
1.0	1.29	1.29	2.32	2.90	1.53
1.2	1.16	1.16	2.17	1.60	1.25

Table 19 : ARL values for positive sided charts for N-Group Runs Chart double exponential distribution (n=5)

Shift $(\theta - \theta_o)$	N-Group Runs UCL=4 L=1	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	32.98	32.77	25.47	32.00	32.00
0.2	6.77	7.45	8.58	10.40	13.83
0.4	2.78	3.17	5.46	5.31	6.58
0.6	1.70	1.96	3.20	3.30	3.51
0.8	1.31	1.52	2.62	2.38	2.16
1.0	1.14	1.29	2.32	2.90	1.53
1.2	1.06	1.16	2.17	1.60	1.25

Table 20 : ARL values for positive sided charts for N-Group Runs Chart Cauchy distribution (n=5)

Shift $(\theta - \theta_o)$	N-Group Runs UCL=10 L=8	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	36.56	32.77	25.47	32.00	32.00
0.2	3.25	3.70	5.65	5.60	29.10
0.4	1.84	1.92	3.61	2.73	26.93
0.6	1.50	1.53	2.96	2.03	24.01
0.8	1.35	1.36	2.71	1.71	22.05
1.0	1.26	1.26	2.56	1.52	19.31
1.2	1.21	1.22	2.46	1.42	17.38

Table 21 : ARL values for positive sided charts for N-Group Runs Chart Cauchy distribution (n=5)

Shift ($\theta - \theta_o$)	N-Group Runs UCL=4 L=1	Synthetic UCL=10 L=4	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	33.16	32.77	25.47	32.00	32.00
0.2	3.45	3.70	5.65	5.60	29.10
0.4	1.71	1.92	3.61	2.73	26.93
0.6	1.33	1.53	2.96	2.03	24.01
0.8	1.19	1.36	2.71	1.71	22.05
1.0	1.13	1.26	2.56	1.52	19.31
1.2	1.09	1.22	2.46	1.42	17.38

Table 22 : In control ARL values for positive sided chart for various values of UCL and L for a nonparametric Shewhart-type synthetic control chart (For n=10)

	<----- L ----->									
UCL	1	2	3	4	5	6	7	8	9	10
1	4	2.67	2.29	2.13	2.06	2.03	2.02	2.01	2	2
2	4.71	3.06	2.57	2.37	2.27	2.22	2.2	2.19	2.18	2.17
3	4.71	3.06	2.57	2.37	2.27	2.22	2.2	2.19	2.18	2.17
4	5.59	3.55	2.93	2.66	2.53	2.46	2.42	2.39	2.38	2.37
5	5.59	3.55	2.93	2.66	2.53	2.46	2.42	2.39	2.38	2.37
6	6.75	4.18	3.39	3.03	2.85	2.75	2.69	2.65	2.63	2.62
7	6.75	4.18	3.39	3.03	2.85	2.75	2.69	2.65	2.63	2.62
8	8.27	5.01	3.98	3.51	3.26	3.12	3.03	2.97	2.94	2.92
9	8.27	5.01	3.98	3.51	3.26	3.12	3.03	2.97	2.94	2.92
10	10.24	6.07	4.74	4.12	3.78	3.58	3.45	3.37	3.31	3.28
11	10.24	6.07	4.74	4.12	3.78	3.58	3.45	3.37	3.31	3.28
12	12.91	7.5	5.76	4.93	4.47	4.18	4	3.88	3.79	3.74
13	12.91	7.5	5.76	4.93	4.47	4.18	4	3.88	3.79	3.74
14	16.51	9.41	7.11	6	5.37	4.98	4.72	4.54	4.41	4.32
15	16.51	9.41	7.11	6	5.37	4.98	4.72	4.54	4.41	4.32
16	21.47	12.04	8.95	7.45	6.59	6.04	5.67	5.41	5.22	5.08
17	21.47	12.04	8.95	7.45	6.59	6.04	5.67	5.41	5.22	5.08
18	28.44	15.69	11.5	9.45	8.26	7.49	6.96	6.58	6.31	6.1
19	28.44	15.69	11.5	9.45	8.26	7.49	6.96	6.58	6.31	6.1
20	38.53	20.95	15.15	12.3	10.62	9.53	8.77	8.22	7.82	7.5
21	38.53	20.95	15.15	12.3	10.62	9.53	8.77	8.22	7.82	7.5
22	52.74	28.32	20.24	16.24	13.88	12.33	11.25	10.46	9.86	9.4
23	52.74	28.32	20.24	16.24	13.88	12.33	11.25	10.46	9.86	9.4
24	74.06	39.31	27.79	22.07	18.68	16.44	14.87	13.71	12.83	12.13

25	74.06	39.31	27.79	22.07	18.68	16.44	14.87	13.71	12.83	12.13
26	106.94	56.19	39.33	30.94	25.94	22.64	20.31	18.57	17.25	16.2
27	106.94	56.19	39.33	30.94	25.94	22.64	20.31	18.57	17.25	16.2
28	155.86	81.18	56.35	43.97	36.58	31.68	28.21	25.62	23.63	22.05
29	155.86	81.18	56.35	43.97	36.58	31.68	28.21	25.62	23.63	22.05
30	233.8	120.9	83.26	64.51	53.29	45.84	40.54	36.59	33.53	31.11
31	233.8	120.9	83.26	64.51	53.29	45.84	40.54	36.59	33.53	31.11
32	360.06	184.9	126.6	97.45	80.01	68.42	60.16	53.98	49.2	45.39
33	360.06	184.9	126.6	97.45	80.01	68.42	60.16	53.98	49.2	45.39
34	566.89	289.5	197.1	151	123.3	104.9	91.77	81.95	74.33	68.24
35	566.89	289.5	197.1	151	123.3	104.9	91.77	81.95	74.33	68.24
36	964.47	490.1	332.1	253.1	205.7	174.18	151.67	134.8	121.7	111.3
37	964.47	490.1	332.1	253.1	205.7	174.18	151.67	134.8	121.7	111.3
38	1679.7	850.2	573.8	435.6	352.7	297.51	258.09	228.6	205.6	187.2
	<----- L ----->									
UCL	1	2	3	4	5	6	7	8	9	10
39	1679.7	850.2	573.8	435.6	352.7	297.51	258.09	228.6	205.6	187.2
40	2890.5	1459	981.7	743.1	600	504.64	436.55	385.5	345.8	314.1
41	2890.5	1459	981.7	743.1	600	504.64	436.55	385.5	345.8	314.1
42	5327.9	2682	1801	1360	1095	918.89	792.99	698.6	625.2	566.5
43	5327.9	2682	1801	1360	1095	918.89	792.99	698.6	625.2	566.5
44	10412	5232	3505	2642	2124	1778.4	1531.8	1347	1203	1088
45	10412	5232	3505	2642	2124	1778.4	1531.8	1347	1203	1088
46	21626	10850	7258	5462	4384	3666.2	3153.1	2768	2469	2230
47	21626	10850	7258	5462	4384	3666.2	3153.1	2768	2469	2230
48	41649	20876	13951	10489	8412	7027.1	6037.9	5296	4719	4258
49	41649	20876	13951	10489	8412	7027.1	6037.9	5296	4719	4258
50	118906	59539	39751	29856	23920	19962	17135	15015	13366	12047
51	118906	59539	39751	29856	23920	19962	17135	15015	13366	12047
52	250000	1E+05	83500	62688	50200	41875	35929	31469	28001	25226
53	250000	1E+05	83500	62688	50200	41875	35929	31469	28001	25226
54	1E+06	5E+05	3E+05	3E+05	2E+05	167084	143286	1E+05	1E+05	1E+05
55	1E+06	5E+05	3E+05	3E+05	2E+05	167084	143286	1E+05	1E+05	1E+05

Table 23 : In control ARL values for positive sided chart for various values of UCL and L for Nonparametric Group Run Control Chart (For n=10)

	<------------- L ----------------->									
UCL	1	2	3	4	5	6	7	8	9	10
1	8	3.56	2.61	2.28	2.13	2.06	2.03	2.02	2.01	2
2	10.21	4.31	3.05	2.59	2.38	2.28	2.23	2.2	2.19	2.18
3	10.21	4.31	3.05	2.59	2.38	2.28	2.23	2.2	2.19	2.18
4	13.22	5.32	3.62	2.99	2.7	2.55	2.47	2.42	2.4	2.38
5	13.22	5.32	3.62	2.99	2.7	2.55	2.47	2.42	2.4	2.38
6	17.55	6.73	4.42	3.54	3.13	2.91	2.78	2.71	2.67	2.64
7	17.55	6.73	4.42	3.54	3.13	2.91	2.78	2.71	2.67	2.64
8	23.79	8.71	5.51	4.29	3.7	3.38	3.19	3.07	3	2.96
9	23.79	8.71	5.51	4.29	3.7	3.38	3.19	3.07	3	2.96
10	32.77	11.51	7.02	5.31	4.47	4	3.72	3.55	3.43	3.36
11	32.77	11.51	7.02	5.31	4.47	4	3.72	3.55	3.43	3.36
12	46.39	15.65	9.23	6.77	5.56	4.87	4.46	4.19	4.01	3.89
13	46.39	15.65	9.23	6.77	5.56	4.87	4.46	4.19	4.01	3.89
14	67.09	21.81	12.44	8.87	7.1	6.1	5.47	5.07	4.79	4.59
15	67.09	21.81	12.44	8.87	7.1	6.1	5.47	5.07	4.79	4.59
16	99.51	31.26	17.29	11.98	9.37	7.87	6.93	6.31	5.88	5.57
17	99.51	31.26	17.29	11.98	9.37	7.87	6.93	6.31	5.88	5.57
18	151.7	46.18	24.81	16.75	12.78	10.51	9.08	8.13	7.46	6.97
19	151.7	46.18	24.81	16.75	12.78	10.51	9.08	8.13	7.46	6.97
20	239.17	70.73	36.99	24.37	18.17	14.63	12.4	10.9	9.84	9.07
21	239.17	70.73	36.99	24.37	18.17	14.63	12.4	10.9	9.84	9.07
22	383	110.43	56.4	36.33	26.52	20.94	17.43	15.06	13.39	12.16
23	383	110.43	56.4	36.33	26.52	20.94	17.43	15.06	13.39	12.16
24	637.36	179.6	89.75	56.62	40.53	31.41	25.69	21.84	19.11	17.11
25	637.36	179.6	89.75	56.62	40.53	31.41	25.69	21.84	19.11	17.11
26	1105.9	305.29	149.56	92.58	65.09	49.57	39.87	33.36	28.76	25.38
27	1105.9	305.29	149.56	92.58	65.09	49.57	39.87	33.36	28.76	25.38
28	1945.8	527.89	254.31	154.9	107.19	80.41	63.74	52.59	44.73	38.96
29	1945.8	**527.89**	254.31	154.9	107.19	80.41	63.74	52.59	44.73	38.96
30	3574.9	955.18	**453.37**	272.1	185.72	137.42	107.49	87.56	73.55	63.29
31	3574.9	955.18	**453.37**	272.1	185.72	137.42	107.49	87.56	73.55	63.29
32	6832.3	1801.8	844.31	**500.5**	337.39	246.67	190.7	153.57	127.56	108.57
33	6832.3	1801.8	844.31	**500.5**	337.39	246.67	190.7	153.57	127.56	108.57
34	13497	3520.7	1632.1	957.3	**638.63**	**462.16**	353.73	282.05	232.02	195.6
35	13497	3520.7	1632.1	957.3	**638.63**	**462.16**	353.73	282.05	232.02	195.6
36	29952	7735.2	3550.7	2062	1362.8	976.89	740.73	**585.21**	**477.05**	**398.59**
37	29952	7735.2	3550.7	2062	1362.8	976.89	740.73	**585.21**	**477.05**	**398.59**
	<------------- L ----------------->									
UCL	1	2	3	4	5	6	7	8	9	10

38	68838	17637	8032.8	4630	3035.8	2159.7	1625.3	1274.6	1031.4	855.46
39	68838	17637	8032.8	4630	3035.8	2159.7	1625.3	1274.6	1031.4	855.46
40	155404	39584	17924	10271	6696.2	4736.8	3544.7	2764.1	2224.3	1834.8
41	155404	39584	17924	10271	6696.2	4736.8	3544.7	2764.1	2224.3	1834.8
42	388900	98571	44414	25327	16432	11568	8615	6685.9	5354.6	4396.2
43	388900	98571	44414	25327	16432	11568	8615	6685.9	5354.6	4396.2
44	1E+06	268243	120394	68388	44198	30994	22994	17777	14183	11600
45	1E+06	268243	120394	68388	44198	30994	22994	17777	14183	11600
46	3E+06	800519	358215	2E+05	130721	91396	67605	52111	41453	33805
47	3E+06	800519	358215	2E+05	130721	91396	67605	52111	41453	33805
48	8E+06	2E+06	953737	5E+05	346724	241961	178638	137439	109125	88823
49	8E+06	2E+06	953737	5E+05	346724	241961	178638	137439	109125	88823
50	4E+07	1E+07	5E+06	3E+06	2E+06	1E+06	851458	653786	518068	420850
51	4E+07	1E+07	5E+06	3E+06	2E+06	1E+06	851458	653786	518068	420850
52	1E+08	3E+07	1E+07	8E+06	5E+06	4E+06	3E+06	2E+06	2E+06	1E+06
53	1E+08	3E+07	1E+07	8E+06	5E+06	4E+06	3E+06	2E+06	2E+06	1E+06
54	1E+09	3E+08	1E+08	6E+07	4E+07	3E+07	2E+07	2E+07	1E+07	1E+07
55	1E+09	3E+08	1E+08	6E+07	4E+07	3E+07	2E+07	2E+07	1E+07	1E+07

From Table 23, we choose the pairs of (L,UCL) for which the ARLs(0) is close to ARL0=380. The following table shows ARLs calculations for various combination of (L, UCL) and shift median for three distributions.

Table 24 : ARL values for positive sided charts for Normal Distribution (n=10)

	UCL=29, L=2		UCL=30, L=3		UCL=31, L=3		UCL=32, L=4		UCL=33, L=4	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	527.87	617.76	453.37	423.04	453.37	463.45	500.49	454.99	500.49	419.18
0.2		31.62		28.93		26.51		28.4		30.93
0.4		5.95		5.32		4.89		5.01		5.03
0.6		2.16		1.98		1.95		2.02		2.03
0.8		1.3		1.29		1.29		1.35		1.34
1		1.08		1.1		1.09		1.13		1.12
1.2		1.02		1.03		1.03		1.03		1.04

	UCL=34, L=5		UCL=35, L=5		UCL=34, L=6		UCL=35, L=6		UCL=34, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	638.63	559.9	638.63	593.58	462.16	447.15	462.16	456.07	353.73	356.05
0.2		33.99		35.04		25.3		25.54		22.94
0.4		5.67		5.85		4.81		4.95		4.47
0.6		2.17		2.16		2.08		2.14		2.04
0.8		1.42		1.43		1.42		1.42		1.4
1		1.16		1.16		1.16		1.15		1.15
1.2		1.08		1.05		1.05		1.05		1.05

	UCL=35, L=7		UCL=36, L=8		UCL=37, L=8		UCL=36, L=9		UCL=37, L=9	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	353.73	346.67	585.21	546.67	585.21	542.1	477.05	477.05	477.05	465.05
0.2		27.95		27.95		34.28		28.16		26
0.4		5.38		5.38		53.81		5		5.21
0.6		2.42		2.42		2.4		2.33		2.32
0.8		1.53		1.53		152		1.53		1.54
1		1.21		1.21		1.22		1.22		1.2
1.2		1.07		1.07		1.08		1.07		1.08

	UCL=36, L=10		UCL=37, L=10	
Shift	Synthetic	GR	Synthetic	GR
0	398.59	438.16	398.59	375.98
0.2		24.31		24.87
0.4		5		4.89
0.6		2.3		2.3
0.8		1.53		1.53
1		1.22		1.21
1.2		1.08		1.08

Table 25 : ARL values for positive sided charts for Cauchy Distribution (n=10)

	UCL=22, L=1		UCL=23, L=1		UCL=24, L=1		UCL=25, L=1		UCL=28, L=2	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	383	372.27	383	378.04	637.36	646.66	637.36	642.82	527.87	538.61
0.2		6.51		6.44		8.09		8.09		5.46
0.4		2.11		2.1		2.33		2.31		1.73
0.6		1.47		1.47		1.56		1.55		1.28
0.8		1.27		1.26		1.31		1.31		1.14
1		1.18		1.18		1.2		1.19		1.09
1.2		1.13		1.12		1.14		1.14		1.06

	UCL=29, L=2		UCL=30, L=3		UCL=31, L=3		UCL=32, L=4		UCL=33, L=4	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	527.87	520.47	453.37	454.56	453.37	451.58	500.49	487.51	500.49	482.34
0.2		5.43		4.57		4.52		4.52		4.51
0.4		1.73		1.63		1.63		1.66		1.66
0.6		1.28		1.25		1.26		1.29		1.29
0.8		1.15		1.14		1.14		1.16		1.16
1		1.09		1.09		1.09		1.11		1.1
1.2		1.06		1.06		1.06		1.07		1.07

	UCL=34, L=5		UCL=35, L=5		UCL=34, L=6		UCL=35, L=6		UCL=34, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	638.63	671.63	638.63	609.84	462.16	496.72	462.16	450.09	353.73	344.4
0.2		4.83		4.84		4.27		4.36		3.9
0.4		1.78		1.79		1.75		1.75		1.74
0.6		1.36		1.36		1.36		1.37		1.36
0.8		1.22		1.22		1.22		1.22		1.22
1		1.15		1.15		1.15		1.15		1.15
1.2		1.11		1.11		1.11		1.11		1.11

	UCL=35, L=7		UCL=36, L=8		UCL=37, L=8		UCL=36, L=9		UCL=37, L=9	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	353.73	361.73	585.21	590.29	585.21	576.22	477.05	482.38	477.05	482.38
0.2		3.98		5.08		4.99		4.67		4.78
0.4		1.74		2.09		2.1		2.07		2.07
0.6		1.36		1.61		1.62		1.62		1.62
0.8		1.22		1.43		1.44		1.43		1.43
1		1.15		1.34		1.33		1.34		1.33
1.2		1.1		1.27		1.28		1.28		1.28

	UCL=36, L=10		UCL=37, L=10	
Shift	Synthetic	GR	Synthetic	GR
0	398.59	405.4	398.59	416.64
0.2		4.53		4.55
0.4		2.08		2.07
0.6		1.61		1.61
0.8		1.44		1.43
1		1.34		1.34
1.2		1.28		1.28

Table 26 : ARL values for positive sided charts for Double Exponential Distribution (n=10)

	UCL=22, L=1		UCL=23, L=1		UCL=24, L=1		UCL=25, L=1		UCL=28, L=2	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	383	383.33	383	379.85	637.36	653.08	637.36	671.26	527.87	526.93
0.2		20.17		19.95		28.47		27.85		19.01
0.4		3.98		4.03		5.01		4.92		3.53
0.6		1.17		1.81		2.03		2.03		1.64
0.8		1.09		1.26		1.33		1.34		1.2
1		1.03		1.09		1.11		1.12		1.07
1.2				1.03		1.04		1.04		1.02

	UCL=29, L=2		UCL=30, L=3		UCL=31, L=3		UCL=32, L=4		UCL=33, L=4	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	527.87	527.7	453.37	406.84	453.37	465.09	500.49	470.74	500.49	485.69
0.2		20.12		15.54		16.3		15.8		15.71
0.4		3.52		3.09		3.11		3.12		312
0.6		1.63		1.57		1.57		1.62		1.62
0.8		1.2		1.2		1.21		1.24		1.24
1		1.07		1.08		1.08		1.1		1.1
1.2		1.02		1.03		1.03		1.04		1.04

	UCL=34, L=5		UCL=35, L=5		UCL=34, L=6		UCL=35, L=6		UCL=34, L=7	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	638.63	658.87	638.63	650.85	462.16	456.98	462.16	505.3	353.73	343.28
0.2		17.68		18.22		14.31		14.32		12.44
0.4		3.38		3.33		3.1		3.11		2.9
0.6		1.73		1.73		1.7		1.71		1.7
0.8		1.31		1.3		1.3		1.3		1.31
1		1.13		1.14		1.14		1.14		1.14
1.2		1.06		1.06		1.06		1.06		1.06

	UCL=35, L=7		UCL=36, L=8		UCL=37, L=8		UCL=36, L=9		UCL=37, L=9	
Shift	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR	Synthetic	GR
0	353.73	333.06	585.21	566.93	585.21	569.36	477.05	468.6	477.05	467.81
0.2		12.2		16.05		16.58		14.31		14.47
0.4		2.96		3.48		3.49		3.36		3.36
0.6		1.71		1.91		1.9		1.89		1.89
0.8		1.3		1.41		1.4		1.41		1.4
1		1.13		1.19		1.19		1.19		1.19
1.2		1.06		1.09		1.09		1.09		109

	UCL=36, L=10		UCL=37, L=10	
Shift	Synthetic	GR	Synthetic	GR
0	398.59	420.97	398.59	387.05
0.2		12.71		13.02
0.4		3.26		3.31
0.6		1.89		1.89
0.8		1.41		1.41
1		1.19		1.19
1.2		1.09		1.09

The results are shown in Tables 27, 28 and 29 for subgroup of size $n=10$ under the normal, the double exponential, and the Cauchy distribution. Due to Bakir (2004), the scale parameter is set to be $\lambda=1/\sqrt{2}$ for the double exponential distribution to achieve a standard deviation of 1.0. For the Cauchy distribution, $\lambda=0.2605$ is chosen a tail probability of 0.05 above $\theta+1.645$, the same as that of a normal distribution with a mean θ and a standard deviation of 1.0. For each chart, the control limits are found such that the in-control ARL is 380 for subgroup of size $n=10$ respectively.

Table 27: ARL values for positive sided charts for Normal distribution (n=10)

Shift ($\theta-\theta_o$)	N-Group Runs UCL=36 L=10	Synthetic UCL=40 L=8	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	438.16	387.32	378.56	344.83	380.00
0.2	24.31	39.11	52.48	88.26	64.67
0.4	5.00	8.19	13.10	25.22	15.73
0.6	2.30	3.19	5.25	9.84	5.37
0.8	1.53	1.84	3.12	4.71	2.51
1.0	1.22	1.36	2.38	2.74	1.55
1.2	1.08	1.15	2.11	1.83	1.18

Table 28 : ARL values for positive sided charts for double exponential distribution (n=10)

Shift (θ- θ_o)	N-Group Runs UCL=36 L=10	Synthetic UCL=40 L=8	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	420.97	387.32	378.56	344.83	380.00
0.2	12.71	21.98	54.87	45.55	82.28
0.4	3.26	5.01	10.50	12.48	21.43
0.6	1.89	2.40	4.47	5.45	7.00
0.8	1.41	1.65	2.94	3.07	2.98
1.0	1.19	1.33	2.38	2.08	1.70
1.2	1.09	1.17	2.16	1.61	1.23

Table 29 ARL values for positive sided charts for Cauchy distribution (n=10)

Shift (θ- θ_o)	N-Group Runs UCL=36 L=10	Synthetic UCL=40 L=8	2-of-2 UCL=7	1-of-1 UCL=15	X-bar UCL=0.833
0.0	405.40	387.32	378.56	344.83	380.00
0.2	4.53	7.05	11.43	14.75	377.48
0.4	2.08	2.64	4.12	4.39	375.07
0.6	1.61	1.88	2.91	2.61	372.65
0.8	1.44	1.64	2.49	1.99	370.24
1.0	1.34	1.47	2.32	1.71	367.83
1.2	1.28	1.38	2.21	1.54	365.42

8.Summary and Conclusion

a) On basis of ATS : A N-GR control chart proposed here performs significantly better than Nonparametric Shewhart Chart and Nonparametric Shewhart's Synthetic Chart for all three distributions. The ATS(δ_1) is significantly less than for the other two charts for all three distributions. Also in steady state N-GR performs better than the remaining two charts. A computer program in open source software R and MATLAB software is used to obtain design parameters of ‘A Nonparametric Group Runs Control Chart’.

b) On basis of ARL : A N-GR control chart proposed here performs significantly better than Nonparametric Shewhart Chart and Nonparametric Shewhart's Synthetic Chart for all three distributions. The ARL(δ_1) is significantly less than for the other two charts for all three distributions. So the proposed N-GR control chart outperforms for all shifts under all the distributions for n=5 as well as n=10. Moreover, the performance of the N-GR is significantly better than the $\overline{X}$ chart for heavy-tailed process distribution like the double exponential and the Cauchy. Also N-GR chart performs better than the Shewhart-type signed-rank chart, Synthetic chart, 1-0f-1 signed-rank chart, 2-of-2 signed-rank chart for a class of continuous symmetric distribution Normal, double exponential and Cauchy.

9.Recommendations for future work.

- Develop Modified Nonparametric Group Runs Control Chart to Detect Increases in Fraction Non Conforming and Shifts in the Process Median.

- Develop A Unit and A Nonparametric Group Runs Control Chart to Detect Shifts in the Process Median.

10.Papers presented and published.

Paper presented ‘A Nonparametric Group Runs Control Chart to Detect Shifts in the Process Median’ at Solapur University, Solapur Sponsored by DST, New Delhi, dated October 19-21, 2013

Paper publish is in process.

11.Bibliography.

Bakir S.T. and ,Reynolds M.R.,(1979) : A nonparametric procedure for process control based on within-group ranking, Technometrics 21:2,175-183.

Bakir S.T.,(2004) : A distribution-free Shewhart Quality Control Chart Based on Signed-ranks, Quality Engineering 16:4,613-623.

Bakir, S.T.(2006) : Distribution-Free Quality Control Charts Based on Signed-Rank-Like Statistics. Communication in Statistics-Theory and Methods, 35, 743-757

Bourke, P.D. (1991), Detecting a shift in fraction nonconforming using run-length control charts with 100% inspection. Journal of Quality Technology, 23:3, 225-238.

Chakraborti, S., Laan P., Bakir S.T. S. (2001) : A Nonparametric Control charts : An overview and Some Results. Journal of Quality Technology 33,:304-314.

Chakraborti, S., Eryilmaz, S. (2007) : A Nonparametric Shewhart-Type Signed-rank Control chart based on runs. Communication in Statistics – Simulation and Computation,36,335-356.

Davis R.B. and Woodall W.H. (2002) : Evaluating and Improving the Synthetic Control Chart. Journal of Quality Technology 34,63-69.

Gadre, M. P. and Rattihalli, R.N. (2004) : A Group Runs control chart for detecting shift in the process mean. Economic Quality Control, 19(1),29-43.

Montgomery, D. C. (2001). Introduction to Statistical Quality control. 4^{th} ed. New York: Wiley.

Pawar, V.V and Shirke, D.T. (2010) : A Nonparametric Shewhart-Type Synthetic Control Chart. Communication in Statistics – Simulation and Computation,39:8, 1493-1505

Wu, Z. and Spedding T. A. (2000) : A Synthetic Control Chart for Detecting Small Shifts in the Process Mean. Journal of Quality Technology 32,32-38.

Wu, Z., Song Yeo and Spedding T. A. (2001): A synthetic control chart for detecting fraction nonconforming increases. Journal of Quality Technology 33:1,104-111.

--0—0--