Anekant Education Society's

Tuljaram Chaturchand College

Department of Mathematics

Class :-Msc 1

Question Bank of Ring Theory :-

Que)Multiple choice questions :-

1) The number of prime ideal of $\mathbb{Z}_{10^5}$ is ……
2) The characteristic of the rings $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$ is
3) Field is a commutative ………ring
4) Every non-zero nilpotent element of the ring R is ……
5) If the ring R has unities e_1 and e_2 , then …..
6) If the integral domain I is of finite characteristic,then I is ………
7) If R is ring in which $a^4 = a$, $\forall a \epsilon R$, then R is ……
8) The Cardinality of an finite integral domain is always …….
9) A skew-field have …divisors.
10) If U is an ideal of the ring R then R/U is a …..
11) If I is an integral domain and $a \neq 0 \epsilon I$ then $a^2 \neq$ ……..
12) $f(x) = x^2 + 8x - 2$ is irreducible over ….
13) Ring of polynomial over a field is a …..
14) If integral domain D is finite characteristic , then its characteristic is ……
15) If U is an ideal of the ring R and $1 \in U$, then ….
16) Let R is commutative ring with unity whose ideals are (0) and R itself , then R is …..
17) A polynomial $f(x)$ *and* $g(x)$ are primitive polynomials then ……is also primitive polynomial.
18) If $f(x)$ *and* $g(x)$ are two non-zero polynomial of $f[x]$ then $\deg(f(x)g(x))$ =…….

19) If f is a homomorphism of a ring R into a ring R′ ,then the set S of all those elements of R which are mapped onto the zero element of R′ is called …….of the homomorphism f.

20) If R is a finite field of characteristic p, then given any $b \in R$, $\exists\ a \in R$ such that …..

Que)Define the following:-

1. Field.
2. Commutataive ring.
3. Ring of Quaternions
4. Subring
5. Subfield
6. Integral Domain
7. Principal Ideal Domain
8. Eculedian Domain
9. Factorization domain
10. Unique Factorization Domain
11. Matrix Ring
12. Ring Homomorphism
13. Kernel of Ring Homomorphism
14. First Isomorphism Thm of Rings
15. Ideal
16. Left ideal
17. Right ideal
18. Prime ideal
19. Maximal ideal
20. Quotient ring
21. Ring of Fractions
22. Polynomial ring
23. Eisenstein Criterion for irreduciblity
24. Cyclotomic polynomial
25. Boolean ring
26. Module
27. Z-Module
28. R-algebra
29. Endomorphisms
30. Gauss Lemma

Que] Answer in one sentence :-

1. $\mathbb{Z}_{\boldsymbol{n}}[\boldsymbol{i}]$is ring with unity?

2. $\mathbb{Z}_{30}$is integral domain or not ?
3. Give an example of coomutative ring but not unity.
4. Give an example of ring with unity but not commutative.
5. Give an example of commutative ring of order 16
6. $\mathbb{Z}_{29}[i]$is integral domain ?
7. find idempotent elements ofind idempotent elements of$\mathbb{Z}$
8. find idempotent elements of$\mathbb{Z}_{10}$
9. find nilpotent elements of$\mathbb{Z}_8$
10. Find units of $\mathbb{Z}$
11. how many are there idempotent elements in $\mathbb{Q}$
12. sum of two subrings is subring ?
13. find maximal ideal of $\mathbb{Z}_{10}$
14. find maximal ideal of $\mathbb{Q} * \mathbb{Q}$
15. what are the maximal ideals of $\mathbb{Z}$
16. show that {0} is prime ideal of $\mathbb{Z}$
17. Is every prime ideal of R is maximal ideal of R?
18. Every maximal ideal of R is prime ideal ?
19. Find Factor rings of $\mathbb{Q}$
20. Is the polynomial $x^2 + 1$ is irreducible over Q

Que]Answer the following :-

1. Prove that any finite integral domain is field.
2. Let R be a ring then prove that $0a = a0 = 0$ for all a € R.
3. Let R be a ring then prove that $(-a)\ b = a\ (-b) = -(ab)$ for all a, b € R.
4. Let R be a ring then prove that if R has an identity 1 ,then the identity is unique and $-a = -(a)$
5. Let R be a ring then prove that $(-a)\ (-b) = ab$ for all a , b € R.
6. Prove that intersection of any nonempty collection of subrings of a ring is also a subring
7. A ring R is called Boolean ring if a^2 $=$ a for all a €R. Prove that every Boolean ring is Commutataive.
8. Prove that only Boolean ring that is an integral domain is Z/2Z.
9. Prove that $\{(r, r)|\ r$ €$R\}$ is a subring of $R * R$.
10. Let R be an integral domain and let $p(x)$, $q(x)$ be nonzero elements of$R[x]$.Then

$$degree\ p(x)\ q(x) = degree\ p(x) + degree\ q(x)$$

11. Let R be an integral domain and let $p(x)$, $q(x)$ be nonzero elements of R[x].Then the units of R[x] are just the units of R
12. Let R be an integral domain and let $p(x)$, $q(x)$) nonzero elements of R[x].Then R[x] is an integral domain.

13. Let R and S be rings and let $f: R \to S$: RS be a homomorphism then the image of f is a subring of S.
14. Let R and S be rings and let $f: R \to S$ be a homomorphism then the Kernel of f is subring of R.
15. state and prove First Isomorphism theorem of Ring.
16. state and prove Second Isomorphism theorem of rings.
17. State and Prove Fourth Isomorphism Theorem for rings.
18. Prove that $2Z$ and $3\ Z$ are not isomorphic.
19. Prove that the rings Z[x] and Q [x] are not Isomorphic.
20. .Find all homomorphic images of Z.
21. Check which of the following are ideals of the rings Z[x] the set of all polynomials whose constant term is a multiple of 3.
22. Check which of the following are ideals of the rings Z[x] the set of all polynomials whose coefficient of x^2 is a multiple of 3.
23. Check which of the following are ideals of the rings Z[x] the set of all polynomials whose Constant term, Coefficient of x, coefficient of x^2 are zero.
24. Check which of the following are ideals of the rings Z[x] the set of all polynomials whose coefficient Sum is equal to zero.
25. If D be an integer that is not a perfect square in Z prove that there exist map f : $Z[\sqrt{D}] \to S$ which is ring Isomorphism.
26. Prove that if I be an ideal of R then $I = R$ iff I contains a unit.
27. Prove that if R is Commutataive. Then R is a field iff its only ideals are 0 and R.
28. Prove that in a ring with identity every proper ideal is contained in a maximal ideal.
29. Prove that Every ideal in a Eculedian Domain is Principal.
30. Prove that Z[x] is not P. I. D.
31. Prove that R is any Commutataive ring such that the polynomial ring R[x] is a P. I. D. Then R is necessarily a field.
32. Prove that If M is maximal ideal Iff R/M is Field.
33. Prove tht R/P is Integral Domain Iff P is prime ideal.
34. Prove that in an integral domain a prime element is always irreducible.
35. In a P. I. D. a nonzero element is a prime iff it is irreducible.
36. In a U. F. D. is a nonzero element is a prime iff it is irreducible.
37. Prove that Z is U. F. D.
38. Let R be a P. I. D. Then there exists a multiplicative Defekind-Hasse norm on R.
39. The prime number $p \in Z$ divides an integer of the form n^2 +1 iff p is either 2 or p is an odd prime congruent to 1 modulo 4.
40. Let I be an ideal of the ring R and let $(I) = I[x]$ denote the ideal of R[x] generated by I then $R[x]/(I) = (R/I)[x]$.
41. Prove that (x, y) is not a principal ideal in $Q[x, y]$.
42. Let F be a field. The polynomial ring F[x] is Eculedian Domain.
43. Let F be a field. The polynomial ring F[x] is a $P.I.D.$ and a $U.F.D.$. D.

44. be a Unique Factorization Domain iff R[x] is $U.F.D.$
45. Let F be a field and let $p(x) € F[x]$.(x) € F[x].as a factor of degree one iff$p(x)$ has a root in F, i. e. there is an $\alpha € F$ with $p(\alpha) = 0$.
46. Prove that a polynomial of degree two or three over a field F is reducible Iff it has a root in F.
47. State and Prove Eisenstein Criterion.
48. Prove that p be a prime. Then multiplicative group $(Z/pZ)^\wedge *$ of nonzero ressidueclasses mod p is cyclic.
49. Prove that if R is a Notherian ring then so is a polynomial ring R[x].
50. Prove that Let R be a ring and let M be an R-module. A subset N of M iff N is nonempty and$x + ry € N$ for all $r € R$ and for all $x , y € N$.
51. Show that (Z,+,•) *is a ring*
52. Show that (Z,+,•) *is a ring*
53. Show that (Z[i],+,•) *is a ring*
54. Show that (M_n(R),+,•) *is a ring*
55. If R is an integral domain then R has exactly two idempotent element.