

Anekant Education Society' s

Tuljaram Chaturchand College, Baramati

Department of Mathematics

Class: M.Sc -I

Question Bank of Partial Differential Equations :-

Que) multiple choice questions :-

1. The integral surface of the partial differential equation $xp+yz=0$ satisfying the condition $u(1,y)=y$ is given by.....
2. General solution of equation $xu_x + yu_y = 0$ is
3. General solution of equation $y' + y = f(x)$ is.....
4. The initial value problem $xy' = y, y(0) = 0, x \geq 0$ has
5. Complete integral of $pde z = px + qy + \log pq$ is
6. Complete integral of $pde z = px + qy + \sin pq$ is
7. Complete integral of $pde f = zpq - p - q$ is
8. Orthogonal trajectory of the family $x^2 + y^2 = C$ are given by
9. Orthogonal trajectory of the family $xy = C$ are given by.....
10. The integrating factor for $y dx - x dy = 0$ is
11. If $f(x)$ and $g(y)$ are arbitrary functions, then the general solution of the partial differential equation $ur-pq=0$ is given by....
12. It is required to solve the Laplace equation $s+r=0, 0<x<a, 0<y<b$, satisfying the boundary conditions, $u(x, 0) = 0, u(x, b) = 0, u(0, y) = 0$ and $u(a, y) = f(y)$. If cn' s are constants, then the equation and the homogeneous boundary conditions determine the fundamental set of solutions of the form.....
13. Suppose $u(x,y)$ satisfies Laplace's equation $\Delta^2 u=0$ in R^2 and $u=x$ on the unit circle. Then, at the origin.....
14. The characteristic curves of the partial differential equation $(2x+u)u_x + (2y+u)u_y = u$, passing through $(1,1)$ for any arbitrary initial values prescribed on a non-characteristic curve are given by.....
15. Complete integral for the pde $z=px+qy-\sin(pq)$ is.....
16. Pick the region in which the following differential equation is hyperbolic
$$y_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y \text{ if } xy \dots$$
17. The Cauchy problem $u_x - u_y = 3$ with the Cauchy data on $\Gamma (s, -s, 2s)$ has..... Solution
18. Which of the following, concerning the solution of the Neumann problem for Laplace's equation, on a smooth bounded domain, is true?

(A) Solution is unique

- (B) Solution is unique upto an additive constant
- (C) Solution is unique upto a multiplicative constant
- (D) No conclusion can be drawn about uniqueness

19. Which of the following is elliptic?

- (A) Laplace equation
- (B) Wave equation
- (C) Heat equation
- (D) $r + 2s - 4t = 0$

20. The general integral of the partial differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$ is....

21. Using the transformation $uy = w$ in the partial differential equation $xp = u + yq$, the transformed equation has a solution of the form w equals.....

22. A partial differential equation requires.....

- A) exactly one independent variable
- B) two or more independent variables
- C) more than one dependent variable
- D) equal number of dependent and independent

23. For partial differential equation, if $b^2 - 4ac = 0$ then equation is called

- A) hyperbolic
- B) parabolic
- C) elliptic
- D) None of these

24. Boundary condition which include direct boundary value is

- A) Dirichlet boundary condition
- B) Neumann boundary condition
- C) forced boundary condition
- D) discrete boundary condition

25. . general solution of equation $u_{xx} + u_{yy} = 0$ is

Que] Answer the following :-

- 1) Eliminate the arbitrary constants a, b from equation $z = (x + a)(y + b) = (x+a)(y+b)$ and find the corresponding P.D.E.
- 2) Eliminate the arbitrary constants a, b from equation $2z = (ax + y)^2 + bz = ax + y^2 + b$ and find the corresponding P.D.E.
- 3) Eliminate the arbitrary constants a, b from equation $z = (xa + by)(xa + by)$ and find the corresponding P.D.E.
- 4) Eliminate the arbitrary constants a, b from equation $z^2(1 + a^3) = 8(x + ay + b)^3$ and find the corresponding P.D.E.
- 5) Eliminate the arbitrary function F from equation $F(z - xy, x^2 + y^2) = 0$ and find the corresponding P.D.E.
- 6) Eliminate the arbitrary function F from equation $F(xy, x + y + z) = 0$ and find the corresponding P.D.E.
- 7) Eliminate the arbitrary function F from equation $F(x + y, x - \sqrt{z}) = 0$ and find the corresponding P.D.E.
- 8) Eliminate the arbitrary function F from equation $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ and find the corresponding P.D.E.
- 9) Eliminate the arbitrary function F from equation $F(zxy, x + y + z) = 0$ and find the corresponding P.D.E.
- 10) Find the general solution of $xp + yq = z$
- 11) Find the general integral of $yzp + xzq = xy$
- 12) Find the general solution of $x(y^2 - z^2)p - y(x^2 + z^2)q = (x^2 + y^2)z$
- 13) Find the general integrals of $z(xp - yq) = y^2 - x^2$
- 14) Find the general integrals of $x(z - 2y) = y^2p - xyq$
- 15) Find the general integrals of $x(y - z) = (z^2 - 2yz - y^2)p + x(y + z)q$
- 16) Find the general integrals of $z = (y + 1)p + (x + 1)q$
- 17) Find the general integrals of $x(y - z)p + yq(z - x) = z(x - y)$
- 18) Find the general integrals of $(x^2 + y^2)p + 2xyq = (x + y)z$
- 19) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(1 + yz)dx + z(z - x)dy - (1 + yx)dz = 0$
- 20) Prove that If $\vec{X} \cdot \text{curl } \vec{X} = 0$ where $\vec{X} = (P, Q, R)$ and μ is an arbitrary differential function of x, y and z then $\vec{\mu X} \cdot \text{curl } (\vec{\mu X}) = 0$
- 21) State and Prove Necessary and sufficient condition of Pfaffian differential equation to be integrable .

- 22) Show that the following Pfaffian Differential equation is integrable $y dx + x dy + 2z dz = 0 + 2z dz = 0$
- 23) integral of $yz dx + 2xz dy - 3xy dz = 0$
- 24) hat the Pfaffian differential equation is integrable and find the corresponding integral $(y^2 + yz)dx + (x^2 + xz)dy + (y^2 - yx)dz = 0$
- 25) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(1 + yz)dx + x(z - x)dy - (1 + yx)dz = 0$
- 26) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(yz)dx + xz dy + (yx)dz = 0$
- 27) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(yz)dx + (x^2y - xz)dy + (x^2z - xy)dz = 0$
- 28) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(6x + yz)dx + (xz - 2y)dy + (2z + yx)dz = 0$
- 29) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $(2x + y^2 + 2xz)dx + 2yxdy + x^2dz = 0$
- 30) Verify that the Pfaffian differential equation is integrable and find the corresponding integral $z(z - y)dx + z(x + z)dy + x(x + y)dz = 0$
- 31) Write down necessary and sufficient condition for integrability of two functions f and g.
- 32) Find the complete integral of $f = z^2 - pqxy = z^2 - pqxy$ by Charpit's method
- 33) Find the complete integral of $f = (p^2 + q^2)y - qz = 0$
- 34) Find the complete integral of $f = (x^2p^2 + q^2y^2) - 4 = 0$
- 35) Find the complete integral of $f = (p^2 + q^2 + 1)z^2 = 1$
- 36) Find the complete integral of $f = p + q - pq = 0$
- 37) Find the complete integral of $f = zpq - p - q$
- 38) complete integral of $f = (p^2 + q^2) = x + y$
- 39) Find the complete integral of $z = px + qy + \log pq$
- 40) Find the complete integral of $z = px + qy + pq$
- 41) Find the complete integral of $(q^2p^2 + x^2y^2) = x^2q^2(x^2 + y^2)$
- 42) Find the complete integral of $(xp^2 + q^2y) = z$
- 43) Find the complete integral of $(2z + p^2 + qy + 2y^2) = 0$
- 44) Find the complete integral of $zy = pxy + qy + pq$
- 45) Solve the following equation by Jacobi method $z + 2u_z - (u_x + u_y)^2 = 0$
- 46) Solve the following equation by Jacobi method $u_z + u_y^2 + u_x^2 = 1$
- 47) Solve the following equation by Jacobi method $xu_x^2 - au_z^2 - u_y^2 = 0$
- 48) Solve the following equation by Jacobi method $xu_x + yu_y = u_z^2$
- 49) Solve the following equation by Jacobi method $p^2x + q^2y = z$
- 50) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$
- 51) Find the integral surface of the equation $x^3p + y(y + 3x^2)q = z(2x^2 + y)$
- 52) Find the complete integral of $x(p^2 + q^2) = pz$
- 53) Find the complete integral of $xp^2 + yq - z = 0$

- 54) Find the complete integral of $q^2 + 2xz = x(yq + xp)$
- 55) Reduce the following into canonical forms and solve if possible $u_{xx} + u_{yy} + 17u_{zz} = 0$
- 56) Reduce the following into canonical forms and solve if possible $u_{xx} + xu_{yy} = 0$ in the region $x < 0 < 0$
- 57) Reduce the following into canonical forms and solve if possible $u_{xx} - (\sec h^4 x)u_{yy} = 0$
- 58) Reduce the following into canonical forms and solve if possible $u_{xx} - 4x^2u_{yy} = \frac{1}{x}u_x$
- 59) Reduce following into canonical forms and solve if possible $x^{14}u_{xx} - 36u_{yy} + 7x^{13}u_x = 0$
- 60) Derive d'Alembert's solution for vibration of infinite string.
- 61) State and prove necessary and sufficient condition for the Neumann problem
- 62) Prove that the solution of the Neuman problem is unique up to the addition of a constants
- 63) write down method to solve Dirichlet Problem for the Upper Half Plane.
- 64) write down method to solve vibration of Semi-infinite String Problem.
- 65) write down method to solve vibration of finite String Problem.
- 66) write down method to solve Neumann Problem for the Upper Half Plane
- 67) write down method to solve Dirichlet Problem for a circle.
- 68) write down method to solve Dirichlet Problem for a rectangle.
- 69) write down method to solve Heat conduction of infinite rod problem.
- 70) write down method to solve Heat conduction of finite rod problem.
- 71) write down method to solve Wave Equation.
- 72) Reduce into Canonical form $(n - 1)^2u_{xx} - y^2u_{yy} = ny^{2n-1}u_y$ where n is any integer
- 73) Reduce into Canonical form $u_{xx} + x^2u_{yy} = 0$
- 74) Solve $u_t = u_{xx}$, $0 < x < l, t > 0$,

$$u(0, t) = u(l, t) = 0,$$

$$u(x, 0) = u(l - x), 0 \leq x \leq l$$

- 75) Solve $u_{tt} - c^2u_{xx} = F(x, t)$, $0 < x < l, t > 0$,

$$u(x, 0) = f(x), 0 < x < l$$

$$u_t(x, 0) = g(x), 0 < x < l$$

$$u(0, t) = u(l, t) = 0, t > 0$$