Anekant Education Society's

Tuljaram Chaturchand College of

Arts, Science and Commerce, Baramati

(Autonomous)

QUESTION BANK

FOR

F.Y.B.Sc SEM-II

STATISTICS

PAPER-II: STAT1202

DISCRETE PROBABILITY AND PROBABILITY DISTRIBUTIONS-II

(With effect from June 2019)

Paper – II (STAT1202)

Discrete Probability and Probability Distributions-II

Unit-1 Introduction to R-Software

A) Questions for 1 mark

I. Choose the correct alternative

1. If X→B(n=5,p=0.4), to find P(X=3) we use command

   a) d b i n o m (5,3,0.4) b) d b i n o m (3,5,0.4)

   c) d b i n o m (0.4, 5,3) d) p b i n o m (3,5,0.4)

2. If X→P(m=4.5), to find P(X≤4) we use command

   a) p p o i s (4,4.5) b) r p o i s (4,4.5)

   c) d p o i s (4,4.5) d) p p o i s (4.5,4)

3. To obtain random sample of size 5 from P(m=2) distribution we use command

   a) p p o i s (5,2) b) r p o i s (5,2)

   c) d p o i s (2,4) d) p p o i s (5,2)

4. If X→B(n=10,p=0.25), to find P(X=4) we use command

   a) dbinom(10,4,0.25) b) dbinom(4,10,0.25)

   c) pbinom(10,4,0.25) d) pbinom(4,10,0.25)

5. Transform command is used to

   a) add new records b) add new columns

   c) extracting data d) deletion data

6. If y=c(1,5,1,2,3,7,1,2) then the result of unique(y) command is

   a) 1 5 1 2 3 7 1 2 b) 1 2 3 5 7

   c) 7 5 3 2 1 d) 1 5 2 3 7

7. If y=c(1,5,1,2,3,7,1,2) then the result of unique(y) command is

   a) 1 5 2 3 7 b) 5 3 7

   c) 1 2 d) 1 7

8. If y=c(1,2,3,4,5) then the result of prod(y) command is

   a) 60 b) 120

   c) 15 d) 110

II. State **True** or **False** of the following

1. Transform command is used to extract elements of vector conditionally.
2. Subset command is used to augment two vectors X and Y , each containing same number of elements.
3. seq( ) function is used to generate a vector elements in sequence.
4. rep( ) function is used to generate a vector with repeated elements.
5. c( ) function is used to generate a vector with repeated elements.
6. length( ) function is used to count the no. of elements in vector.
7. R –software is a case sensitive language.

B) Questions for 2 marks

1. Define: Data frames.
2. Define: Summary function from R- Software.
3. Define: fivenum function from R-Software.
4. Define: Resident data sets in R-Software.
5. If X→B(n=5,p=0.4), write R command for computing P(X=3) and P(X<3).
6. Create a vector y of numbers between 1 to 200 which are divisible by 5.
7. Create a data frame containing roll number and marks in two subjects.
8. If X→ B(10, 0.5) write R-command for calculating P(X<=3).
9. Write output for the following R-commands :

```
>x=c(1,5,2,3)
>y=c(6,7)
>z=x + y
>z
```

10. Write a R-command for creating a vector z having even numbers between 1 and 100.

C) Questions for 4 Marks

1. Create a vector 'WEIGHT' containing following weight in Kg of 6 students 56, 59, 76, 87, 54, 77.

   i) Access weight of 2nd and 4th student.

   ii) Create a vector WEIGHT60 whose weight is greater than 60.

1. Create a vector x of 2 elements 3 and 7. Create a new vector y from x containing elements 3, 7, 3, 7, 3,7.

D) Questions for 6 Marks

1. Create a data frame containing seat number and marks in two subjects.

Unit-2. Bivariate Discrete Distribution

A) Questions for 1 mark

III. Choose the **correct** alternative

1. Suppose $\{(x_i, y_j, p_{ij}); i=1,2,\ldots m, j=1,2,..n\}$ be a bivariate probability distribution.

   Then $\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}$ *is*

   a) 0 b) 1 c) lies between 0 and 1 d) Pi.

2. In conditional probability distribution of Ygiven X=x,

   a) X is variable b) X is constant c) Y is variable d) Y is constant.

3. Two discrete random variables X and Y are said to be independent if

   a) $P_{ij} = P_{i.} + P_{.j}$ for all i and j b) $P_{ij} = P_{i.} \times P_{.j}$ for all i and j

   c) $P_{ij} = P_{i.} / P_{.j}$ for all i and j d) $P_{ij} = P_{i.} - P_{.j}$ for all i and j

4. A bivariate random variable is discrete if and only if

   a) at least one of X and Y is discrete b) X is discrete and Y is finite
   c) both X and Y are discrete d) both X and Y are finite

5. Let (X, Y) be a two dimensional discrete random variable and $P(x_i, y_j) = P(X=x_i, Y=y_j)$.

   The function P is called as joint p.m.f. of (X,Y) if

   a) $P(x_i, y_j) \geq 0$ and $\sum_i P(x_i, y_j) = 1$ b) $P(x_i, y_j) \geq 0$ and $\sum_i \sum_j P(x_i, y_j) = 1$

   c) $P(x_i, y_j) > 0$ and $\sum_i \sum_j P(x_i, y_j) = 1$ d) $P(x_i, y_j) \geq 0$ and $\sum_i \sum_j P(x_i, y_j) < 1$

6. If X and Y are any two random variables then the cov(aX+b, cY+d) is

   a) cov(X, Y) b) abcd cov(X, Y) c) ac cov (X, Y) d) ac cov(X, Y) +bd

IV. State **True** or **False** of the following

1. Var (X-Y) = var (X) – var(Y) – 2cov (X, Y)
2. Correlation coefficient between (3 – X) and (5 – 3Y) is the same as that between X and Y.
3. Marginal probability distribution of X is a univariate probability distribution.
4. For two discrete random variables X and Y, E(X+Y) = E(X) + E(Y) only if X and Y are independent.
5. In conditional distribution of X given Y=y, X is variable and Y is constant.
6. The joint distribution function of (X,Y) is a nondecreasing function in each of the variables X and Y.

B) Questions for 2 marks

1. Define a bivariate discrete random variable.
2. Define Joint probability mass function of (X, Y).
3. Show that when X and Y are independent, the conditional distribution of X given Y = yj, is the marginal distribution of X.
4. Define independence of two discrete random variables.
5. Give two practical situations where use of bivariate random variable is needed
6. Define Expectation of function of bivariate discrete r.v.
7. Define conditional expectation of X given Y=y
8. Define conditional expectation of Y given Y=y
9. Define conditional Variance X given Y=y
10. Define conditional Variance Y given X=x
11. Define conditional expectation of X given Y=yi and conditional expectation of Y given X=xi. Also give their interpretation
12. Define covariance of discrete bivariate r.v.
13. Define correlation coefficient between X and Y
14. Define raw moments in bivariate discrete r.v.
15. Define Central moments of bi-variate discrete r.v.
16. Prove that Cov (X,X) = V(X)
17. Show that Cov (X, -Y) = - Cov(X,Y)
18. Show that Cov (-X, -Y) = Cov(X,Y)
19. If (X, Y) is a discrete bivariate random variable with p.m.f.

$$P(x, y) = \frac{k}{x+y}, \quad x = 0,1$$
$$y = 1,2$$

Find value of k.

C) Questions for 4 marks

1. Show that the conditional probability mass function of X given Y=yj and conditional probability mass function of Y given X = xi satisfy the conditions of probability mass function.
2. Show that when X and Y are independent, the conditional distribution of X given Y = yj, is the marginal distribution of X.
3. Define the joint distribution function of a two dimensional discrete random variable. Also state its properties.
4. Let a fair coin be tossed three times. If X and Y denote number of heads and number of runs respectively, obtain joint probability distribution of X and Y.
5. A bag contains 2 white, 3 red and 5 green marbles. Three marbles are drawn at random from this bag without replacement .Obtain the joint probability distribution of the number of white marbles (X) and the number of red marbles (Y) so drawn.

6. The joint p.m.f. of (X,Y) is

$$P(x, y) = \frac{2x+3y}{72} \qquad x = 0, 1, 2 \quad , \quad y = 1, 2, 3$$

$= 0$ otherwise

Find conditional p.m.f. and expectation of X given Y = 2.

7. A bivariate discrete r.v. (X,Y) has joint probability distribution as follow:

Y \ X	-1	0	2
-1	0.1	0.2	0.3
0	0.1	0.1	0.2

Find conditional expectation of X given Y= 0

8. The joint p.m.f.of (X,Y) is given by

$$P(x,y) = k(x^2 + y^2) \quad ; k > 0 \quad x = -1, 1 \quad y = -2, 2$$

$= 0$ otherwise

(i) Find k

(ii) Obtain marginal probability distributions of X and Y.

(iii) Are X and Y independent?

9. Two balls are selected at random without replacement from a bowl containing 4 blue, 1 red and 2 black balls. Let X denote number of red balls drawn, Y denote number of black balls drawn. Find the joint probability distribution of (X,Y).

10. The joint probability distribution of (X,Y) is given below:

X \ Y	0	1	2
-1	5k	k	3k
0	4k	3k	4k
1	2k	3k	2k

Find (i) value of k

(ii) conditional distribution of X given Y = 2

(iii) conditional distribution of Y given X = 0.

(iv) $P(X>0, Y<2)$

11. X and Y are independent r.v.s. with a joint p.m.f. partly shown in the following table. Find missing probabilities. Also obtain marginal p.m.f.'s of X and Y.

X \ Y	0	1	Total
0	$\frac{1}{6}$	-	-
1	-	-	$\frac{2}{3}$
Total	-	$\frac{1}{2}$	1

12. With usual notation show that E(X+Y) = E(X)+E(Y)
13. With usual notation show that E(XY) = E(X)*E(Y), If X and Y are independent.
14. Give example of joint probability distribution of (X, Y), where E (X*Y) = E(X)*E(Y) holds but X and Y are not independent.
15. Does Cov (X, Y) = 0 imply that X and Y are independent? Justify
16. Prove that Cov (aX, bY) = a.b.Cov (X, Y) where a and b are constant
17. Prove that Cov (X+c, Y+d) = Cov (X, Y) Where c and d are constants.
18. Prove that $V(aX - bY) = a^2V(X) + b^2V(Y) - 2ab.Cov(X, Y)$
19. Prove that V (X + Y) = V (X) + V (Y) + 2 Cov (X,Y)
20. For X and Y two independent discrete r.v.s, prove that V (X + Y) = V (X) + V (Y)
21. For X and Y two independent discrete r.v.s, prove that V (X - Y) = V (X) – V (Y)
22. Define correlation coefficient between two discrete r.v X and Y. Give the interpretation of the various values of correlation coefficient.

23. Prove in case of two discrete r.vs independence $\Rightarrow$ Uncorrelatedness.

24. Prove in case of two discrete r.vs Uncorrelatedness does not imply independence.

25. With usual notation show that $\mu'_{10} = E(X)$, $\mu'_{01} = E(Y)$ and $\mu'_{11} = E(XY)$

26. With usual notation show that $\mu_{20} = V(X)$; $\mu_{02} = V(Y)$.

27. With usual notation show that Corr(X,Y) $=\mu_{11}/\sqrt{\mu_{20} * \mu_{02}}$

28. X and Y are two independent discrete rv.s with $\sigma_x = 3$ and $(2X + 3Y) = 72$ compute σ_Y.

29. Suppose X_1, X_2, X_3 are three discrete r.v.s. with means 2,4 and 5 find i) $E(2X_1+X_2+X_3)$ ii) SD of $(X_1+X_2+X_3)/3$

30. Prove that $V(X - Y) = V(X) + V(Y) - 2 \, Cov(X,Y)$

31. Prove that $COV(X, X+Y) = V(X) + COV(X, Y)$

32. For (X, Y), a bivariate discrete r.v. $\sigma_X = 9$, $\sigma_Y = 4$, Cov (X, Y) = 4 find V (2X – 3Y)

33. For (X, Y) a bivariate discrete r.v. such that Cov (X, Y) = 4 Find i) Cov (2X, 3Y) ii) Cov (X+5, 2Y – 6)

34. Does Cov (X, Y) = 0 imply that X and Y are independent? Justify

35. Suppose X and Y are two discrete rvs, prove that $V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab.Cov(X, Y)$

36. Define correlation coefficient between two discrete r.v X and Y. Give the interpretation of the various values of correlation coefficient.

37. For two discrete rv s X and Y, V (X) = V (Y) = 1,Cov (X, Y) = 1/ 2. find V (3X – 4Y).

38. For two discrete rvs X and Y, V (X) = V (Y) = 1, Cov (X, Y) = 1/ 2, find Corr{(X+5)/2 , (Y-6)/3}

39. For two discrete rvs X and Y, V(X)=V(Y)=1,Cov(X,Y)=1/2, find Corr{(X+5)/2,(Y-6)/3}

40. Suppose a fair coin is tossed thrice. Let X denotes the number of heads and Y denotes the number of tails. Obtain the joint probability distribution of (X, Y). Also obtain the marginal distributions of X and Y.

41. For the following probability distribution

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Find E (2X – Y)

42. Let (X, Y) have the following joint pmf

X \ Y	-1	0	1
0	0	1/6	0
1	1/6	0	1/6
2	0	1 /2	0

Show that COV (X, Y) = 0

43. Let X and Y denote the sum and difference (absolute) of scores obtained when two fair dice are thrown. Obtain the joint p.m.f. of X and Y. Hence, calculate (i) P (X = Y), (ii) P (X — Y = 2), (iii) P (X = 3).

44. Two fruits are to be selected at random from 4 mangoes, 2 oranges and 3 apples. Let X and Y denote respectively the mangoes and oranges selected. Obtain the joint probability distribution of (X, Y).

D) Questions for 6 marks

1. Let a fair coin be tossed three times. If X and Y denote the number of heads and the number of runs respectively, obtain the joint probability distribution of X and Y. Also obtain marginal p.m.f.'s of X and Y.
2. Give an example of joint probability distribution of (X, Y) where E (XY) = E (X) * E (Y) holds but X and Y are not independent.

1. The joint probability distribution of a bivariate discrete random variable (X, Y) is given below:

X \ Y	-1	0	1	2
-1	3k	0	k	3k
0	6k	0	3k	3k
1	0	0	3k	4k

Find (i) k (ii) $P(X < 0, Y \geq 0)$

(iii) $P(X = Y)$ (iv) $P(X^2 + Y^2 \leq 2)$.

(v) $P(X \leq 0$, Y is odd) (vi) marginal p.m.f. of X

2. The joint probability distribution of (X,Y) is given below:

X \ Y	1	2	3
1	5k	k	3k
2	4k	3k	4k
3	2k	3k	2k

Find (i) value of k (ii) $P(X \leq 2, Y \leq 3)$.

(iii) marginal probability distributions of X and Y

Check whether X and Y independent?

3. The joint p.m.f. of (X,Y) is given by

$P(x,y) = k(2x+5y)$, $k > 0$ $X = 1,2$ $Y = 1,2$

$= 0$ otherwise

Find (i) k (ii) marginal probability distributions of X and Y.

(iii) $P(X+Y \leq 3)$ (iv) conditional probability distribution of X given Y =2

4. The joint probability mass function of (X,Y) is given by

$P(x,y) = \frac{2x+3y}{72}$, x=0, 1,2

y=1,2,3

= 0 otherwise

(i) Are X and Y independent ?

(ii) Find P(XY is odd).

(iii) Find the conditional probability distribution of X given Y = 2.

5. Following is the joint probability distribution of (X,Y)

X \ Y	-2	-1	0	1
-1	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$
0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$
1	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$

(i) Obtain conditional probability distribution of X given Y = -1

(ii) Calculate $P(Y \geq 0 \mid X \geq -1)$

(iii) Calculate F(0, 0).

6. Let (X,Y) be a discrete bivariate random variable with the following joint probability distribution:

X \ Y	0	1	2	3
0	k	3k	2k	4k
1	2k	6k	4k	8k
2	3k	9k	6k	12k

(iv) Find k.

(v) Calculate $P(X \leq 1, Y \geq 1)$ and P(X=Y)

(vi) Are X and Y independent?

7. With usual notation prove that

a) E (X+Y) = E (X) + E (Y)
b) E (XY) = E (X) * E (Y) if X and Y are independent

8. A bivariate discrete r.v. (X,Y) has joint probability distribution as follow:

Y \ X	-1	0	2
-1	0.1	0.2	0.3
0	0.1	0.1	0.2

Find conditional expectation and conditional variance of X given Y= 0.

9. A bivariate discrete r.v. (X,Y) has joint probability distribution as follow:

X \ Y	-1	0	1	2
-1	0.10	0.05	0	0.20
0	0.15	0.25	0.05	0.05
1	0.05	0.10	0	0

Calculate Corr$(\frac{3X+5}{2}, \frac{5-3Y}{2})$.

10. The joint probability distribution of (X, Y) is as given below :

X \ Y	-2	0	2
0	0	0.2	0.1
1	0.3	0.1	0.1
2	0.05	0.05	0.1

Find

i) Marginal probability distributions of X and Y.

ii) E(X), E(Y), E(XY), Corr(X,Y)

11. If X_1, X_2, ….Xn are independent discrete rvs with common variance σ^2, show that

$Var(\bar{X}) = \sigma^2/n$ where $\bar{X} = \sum xi/n$

12. Obtain E (2X – Y) and E (XY) for the following probability distribution

Y / X	0	1	2
1	0.06	0.15	0.09
2	0.14	0.35	0.21

13. Obtain Conditional mean and Variance of X given Y=3 for the following joint probability distribution of (x,Y)

Y / X	1	2	3
0	0.1	0.2	0.3
1	0.1	0.1	0.2

14. The joint pmf of X and Y is

Y / X	0	1	2
-1	1/ 6	0	1/ 2
+1	1/ 4	1/ 3	1/ 6

Find E (Y/X = 1) and V (Y/ X= 1)

15. If X and Y are two discrete rvs with joint pmf P (X, Y) = 1/ 4 X = 0, 1, ; Y= 0, 1
= 0 otherwise.

i) Calculate Corr (X, Y)

ii) Calculate V (X+Y)

iii) Find V (3X – 4Y)

iv) Find Corr (2X, 3Y)

v) E {X/(Y=1)}, V {X/(Y=1)}

Unit-3 . Some Standard Discrete Probability Distributions: (Finite sample space)

A) Questions for 1 mark

I] Choose the correct alternative

1. Suppose X and Y are two independent discrete random variables with parameter n. Then distribution of X+Y is…..
   a) Discrete uniform with parameter 2n   b) Binomial distribution
   c) Discrete uniform with parameter n^2 d) none of the above.
2. .If random variable X has binomial distribution with parameters n and p,then….
   a) mean < variance   b) mean > variance
   c) Mean = variance   d) mean ≤ variance
3. If X→Bin(n,p) and E(X) = 5/3,var(X) = 10/9,then the value of q is……
   a) 1/3   b) 2/3
   c) 1/6   d) 5/6
4. A random sample of 5 lucky winners is to be selected from a group of 10 ladies and 20 men using simple random sampling with replacement.The number of ladies lucky winners will follow:
   a) Discrete uniform distribution   b) Binomial distribution
   c) Hypergeometric distribution   d) Bernoulli distribution
5. If X→Bin(n_1,p) and Y→Bin(n_2,p) and X and Y are independent,then the distribution of X+Y is……..
   a) Bin(n_1+n_2,2p)   b) Bin(n_1+n_2,p)
   c) Bin(n_1+n_2,q)   d) not a binomial distribution
6. If X→Bin(n,1/4),then the probability distribution of Y = n-X is……..
   a) b(n,1/4)   b) b(4n,1)
   c) b(n,3/4)   d) b(2n,1/4)
7. A random variable X →H(N,M,n) when N→∞ and $\frac{M}{N}$ = p,the distribution of X is
   a) b(n,q)   b) b(m,p)
   c) b(m,q)   d) b(n,p)
8. If X follows discrete uniform distribution on 0,1,2,….,n and te mean of the distribution is 6.Hence the value of n is
   a) 6   b) 18
   c) 36   d) 12
9. If X→H(10,6,3),then the second factorial moment of X is
   a)10   b) 6
   c) 2   d) 3

II] State whether each of the following statement is True or False.

1. Mean and standard deviation of discrete uniform distribution are equal.
2. A Bernoulli trial is a random experiment which has only two outcomes.
3. Binomial distribution is negatively skewed, if $p<1/2$
4. Mode of Binomial distribution is unique.
5. Hypergeometric distribution is an approximation to binomial distribution.

B) Questions for 2 marks

1. Give two real life situations where uniform distribution can be realized
2. Find the mean of uniform distribution
3. The p.m.f. of uniform random variable X is p(x) = k for x = 5,6,7,…….,16, k.>0 Find the value of k.
4. Define Bernoulli distribution.
5. Give three examples of real life situations where Bernoulli distribution can be used.
6. Define Binomial distribution.
7. Find mean of binomial distribution
8. Define mode of Binomial distribution
9. Is it possible to haveE (X) = 3 and V(X) = 5? Justify
10. Define Hypergeometric distribution.
11. Mention two practical applications of Hypergeometric distribution.
12. Obtain mean of hyper geometric distribution.
13. If X~B(n, p) with n = 20, E(X) = 8, find parameter p and variance (X).

C) Questions for 4 marks

1. Define a binomial distribution and find its mean.
2. Define Hypergeometric distribution and state its mean.
3. State the p.m.f. of a H(N, M, n) variable. Obtain its mean.
4. State the additive property of binomial distribution.
5. A parcel of 12 books contains 4 books with loose binding what is the prob. That a random selection of 6 books (without replacement) will contain 3 books with loose binding?

D] Questions for 6 marks

1. Obtain mean ,varaiance and μ_3 for a Bernoulli r.v. when is the distribution symmetric?
2. A r.v. X has the following discrete uniform distribution

   $P(X) = \frac{1}{n+1}$; X=0,1,…..,n.Find E(X) and Var(X)
3. Let X→ B(n=8,p=1/4) Find i) P(X=3) ii) P(X<3) iii) P(X≤6)
4. Let X→ B(n1,p) ; Y→B(n2,p) Further X and Y are independent. Obtain the conditional distribution of X given X+Y=n. Identify the distribution.
5. State and prove the recurrence relation for binomial probabilities. what is its use?
6. Obtain mode for B(n,p) variable. Is it unique?
7. State the p.m.f. of a H(N,M,n) variable.obtain its mean and variance.
8. Compute γ_1 for a discrete uniform variable and comment on its value.
9. Of the 100 people in a certain village, 40 always tell the truth and remaining always lie. A sample of 10 persons is chosen from these people. What is the prob. That the sample will contain
   - Three ii) No iii) all liers?
10. A group of 20 cricket players contain 7 Maharashtrians and remaining Non-Maharashtrians.

    An Indian team of 12 players is to be formed i) What will be the average no. of Maharashtrians selected in the team? ii) What is the probability that one fourth of players in the team are Maharashtrians?
11. Seventy percent of the letters received by the popular T.V. program 'Surabhi' are written by ladies. What is the probability that
    i) All the five prizes awarded by Surabhi are bagged by ladies
    ii) Exactly 2 by ladies iii) at most by ladies?
12. Let X ~ B(n, p)
    i) if n= 15 and E(X)= 6 find p and V(X)
    ii) If E(X) =9 and V(X)= 3.6, find n and p
    iii)If p= 0.7 and E(X)= 14, find n and V(X)
    iv) If n= 25 and E(X)=10, find p and V(X).
13. Define Binomial distribution. Give interpretation of a B (n, p) r.v. Also mention three real life situations, where Binomial distribution is applied.
14. Let X ~ B (n, p). Obtain the mean and Variance of X. show that Var(X) < E(X)
15. If X and Y are independent binomial Variation with X ~ B(n, 1/ 2).
    Find P (X+Y=5), P (X+Y≤ 1), P (X+Y/2 /1), P{3(X+Y) ≤6)

16. Suppose that the prob. That a light in a classroom will be burnt out is 1/3. The classroom has in all 5 lights and it is unusable. If number of Lights' burning is less than two. What is the probability that the classroom is unusable on random occasion?

17. Let X and Y be two independent binomial variation with parameters (n_1 =5, p= 0.4) and (n_2= 6, p = 0.4) respectively.

Find i) $P(X+Y = 4)$ ii) $P(X+Y \leq 8)$

iii) $P\{X= 1/(X+Y= 7)\}$ iv) $P\{Y= 2 / (X+Y = 8)\}$

18. Obtain mean, Variance and μ_3 for a Bernoulli r.v. when is the distribution symmetric?

19. Show that product of n independent Bernoulli (p) r.v.s. is a Bernoulli random variable with parameter p^n.

20. For a Bernoulli r.v. X, $\mu_3 = 0.7$ what is Var(X)?

21. A r.v. X follows Bernoulli distribution with $\mu_3 = 0$. what is the value of p?

Unit-4.

Standard Discrete Probability Distribution for Countably infinite sample space: Poisson Distribution

A) Questions for 1 mark

I] Choose the correct alternative

1. If random variable X has Poisson(m) distribution then

   a) mean < variance b) mean > variance

   c) Mean = variance d) mean ≤ variance

2. If X→ Poisson(m) then its m.g.f. is given by

   a) $e^{m(t-1)}$ b) $e^{m(t+1)}$

   c) $e^{m(e^{t-1})}$ d) $e^{(me^{t}-1)}$

3. If X→ Poisson(m) with variance 3 then the third cumulant is

   a) $\sqrt{3}$ b) 3^2 c) 3 d) $\frac{1}{3}$

4. X→Bin(n,p) tends to Poisson(m) distribution if,

   a) n→∞ , p→0 b) n→0 , p→0

   c) n→∞ , p→∞ d) n→0 , p→∞

5. The second central moment of Poisson(m) distribution is

   a) $3m$ b) m c) m^2 d) m^3

6. If X→ Poisson(m=4.5) then its mean and mode is given by

   a) 4.5, 4.5 b) 4.5, 4

   c) 4.5, 3 d) 4.5, 3.5

7. Suppose X_1 and X_2 are two independent Poisson variables with parameters m_1 and m_2 respectively then X_1+X_2 follws Poisson distribution with parameter

   a) $m_1 - m_2$ b) $m_1 + m_2$

   c) $m_1\ m_2$ d) $m_1 \div m_2$

8. Let X→ Poisson(m=2) then variance of X is:

   a) 9 b) 4 c) 2 d) 1.4142

9. Let X→ Poisson(m=9) then s.d. of X is:

   a) 9 b) 4 c) 2 d) 3

10. If X → Poisson(m_1) independent of Y → Poisson(m_2) distribution then conditional distribution of X given X+Y=n is

a) Binomial($n, \frac{m_1}{m_1+m_2}$)   b) Poisson($\frac{m_1}{m_1+m_2}$)

c) Binomial($n, \frac{m_1}{m_2}$)   d) Poisson($m_1 + m_2$)

II] State whether each of the following statement is True or False.

1. Mean and standard deviation of Poisson distribution are equal.
2. Poisson distribution is symmetric.
3. Poisson distribution is unimodal.
4. Poisson distribution satisfies additive property.
5. Poisson distribution is positively skew.
6. Suppose X and Y are two independent Poisson random variables then X-Y is also a Poisson random variable.
7. Poisson distribution is an approximation to binomial distribution.
8. If X → Poisson(m_1) independent of Y → Poisson(m_2) distribution then conditional distribution of given X+Y=n is Binomial($n, \frac{m_1}{m_1+m_2}$)

B) Questions for 2 marks

1. Define Poisson probability distribution
2. State additive property of Poisson distribution.
3. State recurrence relation between the probabilities of Poisson distribution.
4. Give two real life situations where Poisson distribution can be used.
5. If $M_X(t) = e^{2.4(e^{t-1})}$ is m.g.f. of r. v. X then identify the distribution of X.
6. If X → Poisson(m) such that P(X=5)=$\frac{3}{10}$P(X=4), find m.

A) Questions for 4 marks

1. Show that mean and variance of Poisson distribution are same.
2. State p.m.f. of Poisson distribution with parameter m. Give three real life situations where Poisson distribution can be applied.
3. State and prove additive property of Poisson distribution.
4. Obtain recurrence relation between the probabilities of Poisson distribution. State its use.
5. Obtain mode of Poisson distribution with parameter m when i) m is an integer ii) m is not an integer.
6. If $M_X(t) = e^{5(e^{t-1})}$ is m.g.f. of r. v. X then find P(X ≥ 2) and Mode of X.
7. If X → Poisson(m) such that P(X=0)=$\frac{1}{2}$, find E(X) and Var(X).
8. If X → Poisson(m) such that P(X=1)= 2P(X=2), find p.m.f. of X.
9. If X → Poisson(2) independent of Y → Poisson(3), find $P(\frac{X+Y}{2} < 2)$, E(X+Y) and Var(X+Y).
10. Let X_1 and X_2 are two independent Poisson variables with means 2 an 3 respectively. Find i) E(X_1/X_1 +X_2=6) ii) Var(X_1/X_1 +X_2=5)
11. If X → Poisson(m) has two modes at 2 and 3, find the probability that it takes either of the values.
12. If X → Poisson (m) such that P(X=2) = $\frac{3}{4}$ P(X=1). Find P(X=0) and the most probable value of X.
13. In the inspection of tinplate by a continuous electrolytic process 12, imperfections are spotted every hour on an average. Find the probability of spotting.
    (i) one imperfection in 10 minutes.
    (ii) at most 2 imperfections in 15 minutes.
14. In a summer season a truck driver experiences on an average one puncture in 1000 km. Applying Poisson distribution, find the probability that there will be
    (i) no puncture, (ii) two punctures in a journey of 3000 km.
15. A firm has 5 cars which can be hired out in a day. The demands for a car on each day are a Poisson variable with average 2. Calculate the probable number of days in a year on which
    (i) no car is demanded, (ii) the demand cannot be fulfilled.
16. A book contains 400 misprints distributed randomly throughout its 400 pages. What is probability that a page observed at random, contains at least two misprints?

17. A manufacturing concern, employing a large number of workers, finds that over a period of time the average absentee rate is 2 workers per shift. Using Poisson approximation calculate the probability that on a given shift

    (i) exactly 2 workers will be absent.

    (ii) more than 4 workers will be absent.

18. In the inspection of a fabric produced in continuous rolls, the number of imperfections per ten yards is a random variable having a Poisson distribution with m = 2.8. Find probability that (i) 10 yards of the fabric will have 3 imperfections (ii) 20 yards of fabric will have at most 6 imperfections.

B) Questions for 6 marks

1. Determine the first four central moments of Poisson distribution and obtain γ_1. Comment on the nature of the distribution.
2. Obtain m.g.f. of Poisson distribution and hence obtain first four raw moments.
3. Obtain m.g.f. of Poisson distribution. Hence obtain its mean and variance.
4. If X → Poisson(2) independent of Y → Poisson(3), find i) $P(\frac{X+Y}{2} < 2)$ ii) E(X+Y)

   iii) P(2(X+Y)>4) iv) Var(X+Y).

5. If X → Poisson(m_1) independent of Y → Poisson(m_2) distribution then find conditional distribution of X given X+Y=n and hence find E(X/X +Y=n), Var(X/X +Y=n)

****