Anekant Education Society's

Tuljaram Chaturchand College

Department of Mathematics

Class :-Msc 1

Question Bank of Complex Analysis :-

2 Marks que.

1) State de Moivre's formula.
2) Define Radius of convergence of the power series.
3) Define analytic functions.
4) Define Branch of the logarithm.
5) Define Mobius transformation.
6) State Orientation Principle.
7) Define function of bounded variation.
8) State Liouville's theorem.
9) State Fundamental theorem of Algebra.
10) State Maximum Modulus Theorem.
11) Define Simply connected set.
12) State an Open mapping Theorem.
13) Define Goursat's Theorem.
14) Define essential singularity.
15) State Casirti-Weierstrass theorem.
16) State Schwarz's Lemma.
17) State Lebesgue's Covering Lemma.
18) State Heine- Borel Theorem.
19) State Morera's theorem.

20) Define Meromorphic function on G.

21) State Symmetry Principle.

22) Define Meromorphic function.

Long Answer type Que.

1) Prove that a subset of R is connected iff it is an interval.

2) Prove Cantor's theorem.

3) Prove that C is Complete.

4) Prove that Let (X, d) be a complete metric space and let Y a subset of X. Then

(Y, d) is a complete metric space iff Y is closed in X.

5) State and Prove Lebesgue's Covering Lemma.

6) State and prove Heine- Borel Theorem.

7) Prove that Composition of two continuous functions is continuous.

8) Suppose f: X-Y is continuous and X is compact then f is uniformly continuous.

9) State and Prove Weierstrass M-test.

10) Let G be either the whole plane C or some open disk. If u: G $\rightarrow$ R is a harmonic function then prove that u has a harmonic conjugate.

11) If S is a Mobius transformation then Prove that S is the composition of translation, dilation and the inversion.

12) Prove that a Mobius transformation takes circles into circles.

13) Show that a Mobius transformation has infinity as it's only fixed point iff it is a translation.

14) Give the power series expansion of logs about z = i and find its radius of convergence.

15) State and Prove Fundamental theorem of Algebra.

16) State and Prove Identity theorem.

17) State and Prove Maximum Modulus Theorem.

18) Show that if f and g are analytic functions on a region G such that fg is analytic then either f is constant or g=0.

19) State and Prove Morera's theorem.

20) If G is simply connected and $f: G \to C$ is analytic in G then prove that f has a primitive in G.

21) State and Prove Open Mapping theorem.

22) Suppose that $f: G - C$ is analytic and one one; show that $f'(z) = 0$ for any z in G.

23) State and Prove Goursat's theorem.

24) If f has an isolated singularity at a then prove that the point $z = a$ is a removable singularity iff $\text{Lim}(z - a) f(z) = 0$ (z tends to a).

25) Let $z = a$ be an isolated singularity of f and let $f(z) = a_n(z-a)^n$ be its Laurent Expansion in ann(a; 0 , R).Then prove that $z = a$ is a removable singularity iff $a_n = 0$ for $n \leq -1$.

26) State and Prove Casorati Weierstrass Theorem.

27) If $f: G - C$ is analytic except for poles show that the poles of f cannot have a limit point in G.

28) Prove that an entire function has a removable singularity at infinity iff it is a constant.

29) State and Prove Residue Theorem.

30) Prove that a Mobius transformation takes circles onto circles.

31) State and Prove Maximum Modulus theorem(all three versions).

32) State and Prove Schwarz's Lemma.

33) State and prove Symmetry Principle.

34) State and prove Runge's approximation theorem.

35) State and Prove Riemann's theorem on removable singularity.

36) Prove that the Meromorphic function in the extended complex plane are rational functions.

37) Show that $f(z) = |z|^2 = x^2 + y^2$ has a derivative only at the origin.

38) Give the Principle Branch of sqrt of $(1-z)$.

39) Discuss the mapping properties of cosz and sinz.

40) Show that if $f: [a, b] - C$ is a Lipschitz function then f is of bounded variation.

41) Prove that an entire function has a pole at infinity of order m iff it is a polynomial of degree m.

42) Suppose f= u + iv is complex valued function defined on open set Ω If u & v are continuously differentiable and satisfy CR equation on Ω then Prove that f is holomorphic on Ω and f′(z) = $\frac{\partial f}{\partial z}$.

43) If f is holomorphic in the region Ω and f′ = 0 then prove that f is constant.

44) Show that $\int_0^{\infty} \frac{1-cosx}{x^2}$ dx = $\frac{\Pi}{2}$.

45) State and prove Cauchy's Integral formula.

46) Prove that every nonconstant polynomial with the complex coefficient has root in C.

47) Prove that every polynomial p(z) = $a_n z^n + a_{n-1} z^{n-1} + \ldots + a_0$ of degree n ≥ 1 has precisely n roots in C.

48) Suppose that f is holomorphic function in a region Ω that vanishes on a sequence of distinct points with a limit point in Ω then prove that f is identically zero.

49) Suppose f is holomorphic in an open set Ω and K ⊂ Ω is compact then prove that there exists finitely many segments say $\Upsilon_1, \Upsilon_2, \ldots \Upsilon_N$ in Ω – K such that f(z) = $\sum_{n=1}^{N} \frac{1}{2\Pi i} \int_{\Upsilon} \frac{f(\xi)}{\xi - z}$ dξ for all z∈ K.

50) Prove that $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ = Π

51) State and Prove Riemann's theorem on removable singularity.

Short answer Question

1) What is the radius of convergence of $\sum_{n=0}^{\infty} logn^2 z^n$

2) State and prove Cauchy theorem for a disc.

3) Write short note on Keyhole Contours.

4) Evaluate 1) $\int \frac{z^2}{z-1}$ dz on |z| = 1.

2) $\int \frac{e^{2z}}{z^4}$ dz on |z| = 1.

5) Determine the number of zeroes with their multiplicity of the polynomial

i) $z^4 + 3z^3 + 6$; C : |z| = 2

ii) $2z^4 - 2z^3 + 2z^2 - 2z + 9$; C : |z| = 1.

6) If f is a nonconstant holomorphic function in a region Ω then prove that f cannot attain a maximum in Ω.

7) Evaluate $\int \frac{5z-2}{z(z-1)}$ dz on C : |z| = 2.

MCQ Que.

1) Let f : C → C be a complex valued function given by

f(z) =u(x, y) + iv(x, y). Suppose that v(x, y) = 3xy^2. Then
(i) f cannot be holomorphic on C for any choice of u
(ii) f is holomorphic on C for a suitable choice of u

(iii) f is holomorphic on C for all choice of u
(iv) v is not differentiable as a function of x and y.

2) The function f(z) = |z|^2, z ∈ C is:
(i) continuous and differentiable everywhere
(ii) everywhere continuous but nowhere differentiable
(iii) continuous everywhere but differentiable only at the origin
(iv) neither continuous nor differentiable anywhere.

3) Let f : C → C be a complex valued function if f(z) and f(z) are both analytic ,then:
(i) f(z) is a constant function
(ii) f(z) is the identity function
(iii) f(z) is unbounded
(iv) f(z) is a non constant entire function.

4)Consider the statements :
(a) if a function is analytic in a bounded domain , then it is bounded
(b) if u(x, y) is harmonic in a domain D, then there exist a harmonic function v(x, y) such that u(x, y) + iv(x, y) is analytic then :
(i) both (a) and (b) are true
(ii) both (a) and (b) are false
(iii) only (a) is true

(iv) only (b) is true

5) Let f be real valued harmonic function on C that is, f satisfies the equation

$\partial^2 f/ \partial x^2 + \partial^2 f/\partial y^2 = 0$. Define the function g = ∂f /∂x - i∂f /∂y, h =∂f /∂x + i∂f /∂y. Then

(i) g and h are both holomorphic function.
(ii) g is holomorphic but h need not be holomorphic.

(iii) h is holomorphic but g need not be holomorphic.
(iv) both g and h are identically equal to the zero function

6) If f is holomorphic and non constant in a region then f is . . .

i) closed ii) neither closed not open

iii)open iv) all of the above.

7) If f is entire and bounded then f is . . .

i) non constant ii) constant iii) open iv) none of the above.