Anekant Education Society's

TuljaramChaturchand College, Baramati

Department of Mathematics

Class: M.Sc –I

Question Bank

Title of Paper : Ordinary Differential Equation **Sub Code:**MAT4105

MULTIPLE CHOICE QUESTION

1) The equation ydx + xdy = 0, is

a) exact D.E.

b) non-exact D.E.

c) partial D.E.

d) None of the above.

2) A solution of a D .E. whichcontains no Arbitrary constants is

a) Particular sol^n

b) general sol^n

c) primitive

d) None of these.

3) Two sol^n y_1 & y_2 are linearly dependent if

a) $W(y_1, y_2) = o$

b) $W(y_1, y_2) \neq o$

c) $W(y_1, y_2) = 1$

d) None of these.

4) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y''(x) + P(x)y' + Q(x)y = 0$ on [a, b]

then for general sol. Of this equation, y_1 & y_2 are

a) linearly dependent

b)) linearly independent

c) proportional

d) None of these.

5) The value of wronskian $W(x, x^2, x^3)$ =

a) $2x^4$ b) $2x^2$ c)) $2x^3$ d) None of these.

SHORT ANSWER QUESTION.

1) Find the solution of initial value problem y”+ y = 0, y(0)=0 & y’(0)=1

2) Prove that if $y_1(x)$ & $y_2(x)$ are any two solutions of y”+P(x)y’+Q(x)y=0 then$c_1y_1(x) + c_2y_2(x)$ is also solution for any constants c_1&c_2.

3) find the general solution of

a) x^2y'' +2xy’-2y=0

b) y”-xf(x)y’+f(x)y=0

c)y”-f(x)y’+[f(x)-1]y=0

4) find y_2 and general solution of each of the following equation from the given solution y_1

a) y”+y=0 , y_1=sinx

b) xy”+ 3y’=0 , y_1=1

c) y”-y=0 , $y_1=e^x$

5) find the solution of the following initial value problem

a) y”-5y’+6y=0 , $y(1)=e^2$& $y'(1) = 3e^2$

b) y”-6y’+5y=0 , y(0)=3 & y’(0)=11

c) y”+4y’+5y=0 , y(0)=1 & y’(0)=

6) Using the transformation $x=e^z$ in Euler’s equidimensional equation solve the following problem

a) $x^2y''+3xy'+10y=0$

b)$x^2y''+2xy'-6y=0$

c) $4xy''^2-3y=0$

7) Consider the two functions $f(x) = x^3$& $g(x) = x^2|x|$ on the interval [-1, 1].

a) Show that their Wronskian W(f, g) vanishes identically.

b) Show that f & g are not linearly dependent.

8) By inspection or experiment find two linearly independent solutions of $x^2y'' - 2y = 0$ on the interval an determine the particular solution satisfying the initial conditions $y(1)= 1$, $y'(1) = 8$.

9) Verify that $y_1 = 1$, $y_2 = \log x$ are linearly independent sol^n. of the equation $y'' + (y')^2 = 0$ on any interval to the right of the origin. Is $c_1+c_2\log x$ the general sol^n?

10) Verify that $y_1 = x^2$ is one sol^n of $x^2y'' + xy' - 4y = 0$. Find y_2& general sol^n.

11) Verify that $y_1 = e^x$ is one sol^n of $xy'' -(2x+1)y'+(x+1)y = 0$. Find y_2& general sol^n.

12) Find the general sol^n of $y'' - \frac{x}{x-1} y' + \frac{1}{x-1}y = 0$ where $y_1 = x$ is one solution.

13) Find the general sol^n of the following equation

a) $y'' + y' - 6y = 0$

b) $y'' + 2y' + y = 0$

c) $y''+ 8y = 0$

d) $2y'' - 4y' + 8y = 0$

14) Find the general solution of each of the following equation:

a) $y''+3y'-10y = 6e^{4x}$

b) $y''+10y'+ 25y = 14e^{-5x}$

c) $y'' - 3y' + 2y = 14\sin 2x - 18\cos 2x$

15) Find the particular solution of

a) $y''- y'-6y = e^{-x}$.

b) $y'' + 4y = \tan 2x$.

16) Find the normal form of the Bessel function $x^2y'' + xy' +(x^2 - P^2)y = 0$.

17) For each of the following differential equations, locate & classify its singular points on the x-axis.

a) $y'' - \frac{2x}{1-x^2} y' + \frac{p(p+1)}{1-x^2} y = 0$

b) $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$

18) Find the Radius of convergence of the following series:

a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ b) $\sum_{n=0}^{\infty} \frac{x^n}{n^n}$

19) Replace each of the following D.E. by an equivalent system of first order equation.

a) $y'' - x^2 y' - xy = 0$

b) $y''' = y'' - x^2 (y')^2$

20) Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ & $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are sol^n of homogeneous system $\frac{dx}{dt} = x + 3y \frac{dy}{dt} = 3x + y$

21) write the general solution of the following:

a) $\frac{dx}{dt} = -3x + 4y \frac{dy}{dt} = -2x + 3y$

b) $\frac{dx}{dt} = 5x + 4y \frac{dy}{dt} = -x + y$

22) Let f (x, y) is a continuous function that satisfies a Lipschitz condition on a strip defined by $a \le x \le b$ & $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip then the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has one & only one solution $y = y(x)$ on the interval $a \le x \le b$.

23) Describe the phase portrait of each of the following system:

a) $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = o \end{cases}$ b) $\begin{cases} \frac{dx}{dt} = -x \\ \frac{dy}{dt} = -y \end{cases}$

24) Show that $F(x, y) = xy^2$ does not satisfy a Lipschitz condition on any strip $a \le x \le b$ & $-\infty < y < \infty$.

25) Write Short note on critical points.

26) Determine wheather each of the following functions are positive definite –ve definite or neither

a) $x^2 - xy - y2$

b) $2x^2 - 3xy + 3y2$

c) $-x^2 - 4xy - 5y2$

ANSWER IN ONE SENTENCE

1) Define Ordinary Differential Equation.

2) Define Linear Differential Equation.

3) what is the order of a Differential Equation.

4) Write down Legendre'sequation and Bessel's equation.

5)Define secondorder linear Homogeneous Differential Equation.

6) Define Wronskian of y_1& y_2.

7) What is Standard form & Normal form of second order linear equation.

8) State Sturm Separation Theorem.

9) State Sturm Comparison Theorem.

10) Define Radius of convergence.

11) Define Ordinary & Singular points.

12) Define Regular Singular point.

13) State Picards theorem

14) State Gauss's Hypergeometric equation.

15) Define Degree of Differential Equation.

16) Define phase portrait

17) Define Isolated Point

18) Define Critical Point

19) Write down the types of critical points.

20) State Lipschitz condition.

LONG ANSWER TYPE QUESTION

1) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y''(x) + P(x)y' + Q(x)y = 0$ on [a, b] then their Wronskian $W(y_1, y_2)$ is either identically zero or never zero on [a, b].

2) If y_1 (x) and $y_2(x)$ are any two solutions of equation $y''(x) + P(x)y' + Q(x)y = 0$ on [a, b] then they are linearly dependent on this interval iff their wronskian is identically zero.

3) Explain the method of the use of known solution to find another solution of given differential equation.

4) a) If n is positive integer ; find two linearly independent solutions of

$xy'' - (x + n)y' + ny = 0$

b) Find the general sol^n of the equation in part a) for the cases n = 1, 2, 3.

5) Explain the method of sol^n of the homogeneous equation with constant coefficients.

6) The equation $x^2y'' + pxy' + qy = 0$ where p & q are constants is called Equidimentional. equation. Show that the change ofindependent variable given by $x = e^z$ transforms it into an equation with constant coefficient.

7) Explain the method of Undetermined coefficients.

8) If k & b are positive constants find the general solution of $y'' + k^2y = \sin bx$.

9) Use the Principle of Superposition to find the general sol^n of the following:

a) $y'' + 4y = 4\cos 2x + 6\cos x + 8x^2 - 4x$

b) $y'' + 9y = 2\sin 3x + 4\sin x - 26e^{-2x} + 27x^3$

10) Explain the method of variation of parameters.

11) Find particular solution of the following equation.

a) $Y'' - 2y' - 3y = 64xe^{-x}$

b) $y'' - 3y' + 2y = (1 + e^{-x})^{-1}$

c) $y'' + y = \sec x$

12) State and prove Sturm Separation Theorem.

13) Prove that if a function $q(x) < 0$, $u(x)$ is nontrivial solution of normal form $u''(x) + q(x)u = 0$ then $u(x)$ has atmost one zero.

14) let $u(x)$ be nontrivial solution of $u''(x) + q(x)u = 0$, where $q(x) > 0 \forall x > 0$ if $\int_1^\infty q(x)dx = \infty$ then $u(x)$ has infinitely many zero's on the x-axis.

15) State & prove Sturm Comparison Theorem.

16) a)Express $\sin^{-1}x$ in the form of $\sum a_n x^n$ by solving $y' = (1 - x^2)^{-1/2}$.

b) Using above series show that

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3.2\text{^}3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{5.2\text{^}5} + \ldots$$

17) Solve the equation $y' = x - y$, $y(0) = 0$ & verify the sol^n by directly solving first order linear equation.

18) Find the series solution of D.E. & radius of convergence of each linearly independent function y_1& y_2.

a) $y'' + y = 0$

b) $y'' - 2xy' + y = 0$

19) Find the power series solution of Legendre's equation

$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$

20) Find the power series solution of following:

i) $y'' + xy' + y = 0$

ii) $(1 + x^2)y'' + 2xy' - 2y = 0$

21) Determine the solution of following using regular singular pts.

i) $2x^2y'' + x(2x + 1)y' - y = 0$

ii) $4xy'' + 2y' + y = 0$

22) Find the initial equation & its roots for the following D.E.

a) $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$

b) $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$

23) Find two independent Frobenius series solution of the following equation.

a) $xy'' + 2y' xy = 0$

24) Determine Gauss's Hypergeometric Equation.

25) Find the general sol of each of the following equation near the indicated singular point.

a) $x(1-x)y'' + (\frac{3}{2} - 2x)y' + 2y = 0$, $x = 0$.

b) $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$, $x = -1$.

26) Let f (x, y) is a continuous function that satisfies a Lipschitz condition on a strip defined by $a \leq x \leq b$ & $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip then the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has one & only one solution $y = y(x)$ on the interval $a \leq x \leq b$.

27) Solve the following initial value problem by Picards method & compare the result with the exact solution $\frac{dy}{dx} = z$, $y(0) = 1$ $\frac{dz}{dx} = -y$, $z(0) = 0$.