Anekant Education Society's

Tuljaram Chaturchand College, Baramati

Department of Mathematics

Class:F.Y.B.Sc. Computer Science

Question Bank

Title of Paper : Algebra **Sub Code:**CSMT1102

Answer in One Sentence (or in 2 – 3 lines) (2 marks questions)

1. State the principle of mathematical induction.
2. Define Cartesian product of sets.
3. True or false: If $|A| = 5$, then $|P(A)| = 25$.
4. Define range of a function.
5. Define a one-one function.
6. Let $f: \boldsymbol{R}^+ \to \boldsymbol{R}^+$ define by $f(x) = x^2$. Does f injective?
7. Define onto function.
8. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ define by $f(x) = x^2$. Does f surjective?
9. Let $f: \boldsymbol{R}^+ \to \boldsymbol{R}^+$ define by $f(x) = x^2$. Does f bijective?
10. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ defined by $f(x) = x^2$. Find $f^{-1}([4, 16])$.
11. Define composition of functions.
12. Define binary operation.
13. Define group.
14. True or false: If $|A| = 4$ and $|B| = 5$, then total number of functions from A to B is 5^4.
15. Define greatest common divisor of integers.
16. Show that there are no two integers a, b such that $(a, b) = 3$ and $a + b = 100$.
17. If $b = aq + r$, $0 \leq r < |a|$, then prove that $(a, b) = (a, r)$.
18. True or false: A number is divisible by 4 if and only if last digit of the number is divisible by 4.
19. Find all primes which divide 50!.
20. If p is prime and $a^2 \equiv b^2 (mod\ p)$, then show that either $p|(a + b)$ or $p|(a - b)$.
21. In $\mathbb{Z}_{12}$, calculate *i*) $(\overline{2} \cdot \overline{9} + \overline{1})^{-1}$, *ii*) $-\overline{5} \cdot (\overline{4} + \overline{5})$.
22. Find all generators of a cyclic group of order 10.
23. Is the set of natural numbers is closed under the substraction?
24. Define cyclic group.
25. Define subgroup.
26. List all the elements in Z_6 which satisfy $x^2 = x$.

27. Give an example of matrix which is in row echelon form.
28. Define homogeneous system.
29. Define row echelon form.
30. Define rank of a matrix.
31. Write augmented matrix for the following system of linear equations
    $x + 2y - 3z + 4w = 2,\ 2x + 5y - 2z + w = 1,\ 5x + 12y - 7z + 6w = 7.$
32. Find the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.
33. Give an example of matrices A and B such that,det(A+B) ≠ det(A)+det(B).
34. Find all solutions of the following system

$$x + 2y = 4\,, 2x + 4y = 8.$$

35. Let D= $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ then compute D^9.

Short Answer Questions (4 marks questions)

1. Prove that the sum of first n positive integers is $\frac{n(n+1)}{2}$.
2. Prove that $3|(n^3 + 2n)$, whenever n is a positive integer.
3. Show that $n^2 < 2^n$ for all positive integers $n \geq 5$.
4. Prove that $n! > 2^n$ for every integer $n \geq 4$.
5. Prove that $2^{2n} - 1$ is divisible by 3.
6. Let A and B be sets. Show that $\overline{A \cap B} = \bar{A} \cup \bar{B}$.
7. Let A and B be sets. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.
8. Let A, B and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.
9. Suppose $B \subseteq C$. Prove that for any set A,

   i) $A \cup B \subseteq A \cup C$.
   ii) $A \cap B \subseteq A \cap C$.

10. Let f be a function from A to B. Let S and T be subsets of B. Show that

$$f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T).$$

11. Let f be a function from A to B. Let S and T be subsets of B. Show that

$$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T).$$

12. If $f: \boldsymbol{R} \to \boldsymbol{R}$ is defined by $f(x) = 2x + 3$, then show that f is bijection and hence find f^{-1}.

13. Let $f: [0,1] \to [a, b]$, $(a < b)$ defined as $f(x) = bx + a(1 - x)$. Show that f is a bijection.

14. Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. List all possible functions from A to B.

15. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined as $f(x) = x^3 + 2$, and $g: \boldsymbol{R} \to \boldsymbol{R}$ be defined as $g(x) = \sqrt[3]{x}$. Find $g \circ f$ and $f \circ g$. Are they same?

16. Let Q be the set of all rational numbers on Q define binary operation * as $a * b = \frac{a+b}{2}$.

    Is * associative? Justify.

17. Show that the set S = { a + b $\sqrt{2}$ / a,b ∈ Z } is a group under addition. Is it an abelian group ? Justify.

18. Show that H = $\{\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} / x \in R\}$ is a subgroup of G = $\{\begin{bmatrix} a & b \\ o & d \end{bmatrix} / ad \neq 0\}$under matrix multiplication.

19. Show that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$.

20. Let $S = R - \left\{\frac{1}{2}\right\}$. Define a binary operation * on S as $a * b = a + b - 2ab$. Show that * is associative. Find the identity element with respect to *.

21. If $a|b$ and $a|c$ then prove that $a|(bx + cy)$ for any integers x and y.

22. Find all integers n such that $n^2 + 1$ is divisible by $n + 1$.

23. If a, b are integers, not both zero, then prove that there are integers x_0 and y_0 such that

$$(a, b) = ax_0 + by_0.$$

24. If $(a, b) = d$, then prove that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

25. Let $a > b$ and $(a, b) = 1$. Show that $(a + b, a - b)$ is either 1 or 2.

26. Prove that if p is a prime and a, b are integers such that $p|ab$, then either $p|a$ or $p|b$.

27. Show that there are infinitely many primes of the form $4n + 3$.

28. Show that there are infinitely many primes of the form $6n-1$.

29. Show that $\sqrt{2}$ is not rational number.

30. Show that $\sqrt{3}$ is not rational number.

31. If $a \equiv b(mod\ n)$ and $c \equiv d(mod\ n)$, then prove that

   i) $(a+c) \equiv (b+d)(mod\ n)$.
   ii) $ac \equiv bd(mod\ n)$.

32. If $ax \equiv bx(mod\ n)$ and $(x,n)=1$, then prove that $a \equiv b(mod\ n)$.

33. Prove that there are precisely n distinct residue classes modulo n.

34. Prepare addition and multiplication table for $\mathbb{Z}_6$.

35. Prepare addition and multiplication table for $\mathbb{Z}_5$.

36. Find the unit digit of 7^{100}.

37. Show that 8n+3 and 5n+2 are relatively prime numbers for n $\in N$.

38. Show that 41 divides $2^{20}-1$.

39. Find the remainder of 7^{486} when divided by 13.

40. Find the remainder of $7^{361}+7^{362}$ when divided by 11.

41. Solve the following system by Gauss elimination method

$$x+y+2z=9,\ 2x+4y-3z=1,\ 3x+6y-5z=0.$$

42. Solve the following system by Gauss elimination method

$$x+y+2z=8,\ -x-2y+3z=1,\ 3x-7y+4z=10.$$

43. Solve the following system by Gauss elimination method

$$2x+y+5z+w=5,\ x+y-3z-4w=-1,\ 3x+6y-2z+w=8,$$

$$2x+2y+2z-3w=2.$$

44. Solve the following system by Gauss Jordan method

$$2x+z=4,\ x-2y+2z=7,\ 3x+2y=1.$$

45. Solve the following system by Gauss Jordan method

$$2x - y - 3z = 0, -x + 2y - 3z = 0, x + y + 4z = 0.$$

46. Solve the following system by Gauss Jordan method

$$2x + y + 5z + w = 5, x + y - 3z - 4w = -1, 3x + 6y - 2z + w = 8,$$

$$2x + 2y + 2z - 3w = 2.$$

47. Show that the following system is inconsistent

$$x + y + z = 3, 2x - y + 3z = 2, 3x - 2y + z = 4, 4x + y + 5z = 2.$$

48. Show that the following system have infinitely many solutions

$$2x + z = 4, x - 2y + 2z = 7, 3x + 2y = 1.$$

49. Reduce the following matrix into row echelon form.

$$A = \begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & 3 & -1 & -5 \\ 4 & -1 & 1 & -1 \\ 5 & -3 & 3 & 1 \end{bmatrix}$$

50. Find the value of λ and μ so that the system given below admits

a) Unique solution b) No solution c) Infinitely many solutions

$$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu.$$

51. Show that the system of equations.

$x + y + 2z = a, x + z = b, 2x + y + 3z = c$ is consistent only if $a + b = c$.

52. Find the inverse by using row reduction method if it exists.

$$B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ -3 & -2 & 3 \end{bmatrix}$$

53. Find Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

54. Reduce the following matrix into reduced row echelon form.

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & -1 & 0 & 2 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

55. Find inverse of $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.

56. Solve the following system by using matrix inversion method.

$$2x + 3y + 4z = 1, 2x + y + z = 2, -x + y + 2z = -2.$$

57. Find column rank,

$$A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 2 & -1 & 3 \end{bmatrix}$$

58. Find row rank,

$$A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 2 & -1 & 3 \end{bmatrix}$$

Long Answer Questions (8 marks questions)

1. Find g.c.d. of 1769 and 2378, and express it in the form
$$(1769,2378) = 1769x_0 + 2378y_0 \text{ for some } x_0, y_0 \in \mathbb{Z}.$$
2. Find g.c.d. of 3997 and 2947, and express it in the form
$$(3997, 2947) = 3997x_0 + 2947y_0 \text{ for some } x_0, y_0 \in \mathbb{Z}.$$
3. Find g.c.d. of 7234 and 3476, and express it in the form
$$(7234, 3476) = 7234x_0 + 3476y_0 \text{ for some } x_0, y_0 \in \mathbb{Z}.$$
4. Find g.c.d. of 5291 and 4514, and express it in the form
$$(5291, 4514) = 5291x_0 + 4514y_0 \text{ for some } x_0, y_0 \in \mathbb{Z}.$$
5. Show that 4999 and 1109 are relatively prime. Also find x_0 and y_0 such that
$$1 = 4999x_0 + 1109y_0.$$
6. Find LU – decomposition of the coefficient matrix and use this decomposition to solve the system.

$$\begin{bmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

7. Solve the linear system AX=B by LU factorization method.

$$8x + 12y - 4z = -36, 6x + 5y + 7z = 11, 2x + y + 6z = 16$$