Department of Mathematics

F.Y.B.Sc. Question Bank

Paper–I: *Geometry*

Answer in One Sentence (or in 2 – 3 lines) (1 or 2 marks questions)

1. Define centre of the conic, and give an examples of a non-central.
2. Find the form of the equation $2x^2 + 3xy - 4y^2 + x + 3 = 0$ when origin is shifted to the point $(-2, 1)$.
3. Shift the origin to the point $(-1, 2)$ and transform the equation $x^2 + y^2 + 2x + 4y = 0$.
4. The origin is shifted to the point $(h, 2)$, find the value of h so that the transformed equation of locus given by $x^2 + 4x + 3y = 5$ will not contain the first degree term in x.
5. The origin is shifted to the point $(h, -1)$, find the value of h so that the transformed equation of locus given by $2x^2 + 4x + 3y = 7$ will not contain the first degree term in x.
6. The origin is shifted to the point $(-2, k)$, find the value of k so that the transformed equation of locus given by $2y^2 + 3x + 4y = 0$ will not contain the first degree term in y.
7. Find the centre of conic $x^2 - 4xy - 2y^2 + 10x + 4y = 0$.
8. Find the centre of conic $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$.
9. Find the centre of conic $3x^2 - 4xy + 6y^2 + 11x - 17y + 13 = 0$.
10. Find the centre of conic $5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$.
11. Discuss the nature of the conic $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$.
12. Discuss the nature of the conic $5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$.
13. Define direction cosines.
14. Define direction ratios.
15. What is relation between direction ratios and direction cosines?
16. Give the condition for two lines to be perpendicular.
17. Give the condition for two lines to be parallel.
18. Find the equation of the plane passing through the point $(2, 3, 5)$ and perpendicular to the line whose direction ratios are $3, -2, 6$.
19. Find the equation of the plane passing through the point $(1, -3, -4)$ and parallel to the plane $6x + 2y - 3z = 5$.
20. Find the equation of the plane passing through the point $(4, 0, -1)$ and parallel to the plane $2x - 5y + \sqrt{7}z + 5 = 0$.
21. Define the normal form of the equation of a plane.
22. Give the formula for angle between two planes.
23. Find the angle between the planes $2x - y + 2z + 1 = 0$ and $3x + 2y + 6z - 5 = 0$.

24. Show that the origin and the point $(2, -4, 2)$ lie on the different sides of the plane $x + 3y - 5z + 7 = 0$.
25. Show that the points $(-2, 2, -1)$ and $(1, -1, 1)$ lie on the different sides of the plane $x - 2y + z + 5 = 0$.
26. Find the distance of the point $(1, 1, 4)$from the plane $3x - 6y + 2z + 11 = 0$.
27. Find the distance of the point $(2, 3, 5)$from the plane $2x + y - z = 4$.
28. Find the distance of the point $(3, -1, 2)$from the plane $5x - 3y + 2z = 6$.
29. Find the joint equation of planes $2x + 3y - z = 0$ and $x - y + 5z = 0$.
30. Find the equation of the line through $(3, 1, 2)$ and perpendicular to the plane $2x - 2y + z + 3 = 0$.
31. Find the angle between the lines whose direction ratios are given by $2, -2, 1$ and $2, 1, -2$.
32. Find the equations of a line passing through the points $(-2, 1, 3)$ and $(3, 1, -2)$.
33. Find k, so that the lines $\frac{x-1}{-3} = \frac{y-1}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other.
34. Give the condition for two lines to be coplanar.
35. Define skew lines.
36. Show that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are coplanar.
37. Show that the lines $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-1}{-1}$ and $\frac{x-4}{2} = \frac{y-2}{-1} = \frac{z-4}{2}$ are coplanar.
38. Show that the lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$ and $\frac{x+1}{4} = \frac{y+1}{4} = \frac{z+1}{-1}$ are coplanar.
39. Show that the line $x + 10 = \frac{8-y}{2} = z$ lies in the plane $x + 2y + 3z = 6$.
40. Define the sphere.
41. Find the equation of the sphere having centre $(2, -3, 4)$ and radius 5.
42. Find the equation of the sphere having centre $(-1, 2, 1)$ and radius 3.
43. Find the equation of the sphere centered at origin of radius 1.
44. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z + 5 = 0$.
45. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2cx - 2cy - 2cz + 2c^2 = 0$.
46. Find the centre and radius of the sphere $3x^2 + 3y^2 + 3z^2 + 6x - 9y - 12z + 15 = 0$.
47. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z + 5 = 0$.
48. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 6x - 4y + 2z + 5 = 0$.
49. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 6x - 4y - 6z - 14 = 0$.
50. Obtain the equation of the smallest sphere passing through the points $(-1, 2, 3)$ and $(1, 3, -4)$.
51. Obtain the equation of the smallest sphere passing through the points $(2, -3, 1)$ and $(-1, -2, 4)$.
52. Obtain the equation of the sphere described on the line join of $(2, -3, 4)$ and $(-5, 6, -7)$ as diameter.
53. Find the equation of the sphere which circumscribe the tetrahedron: $(0, 0, 0), (0, 3, 0), (5, 0, 0), (0, 0, 7)$.

54. Find the equation of the sphere which circumscribe the tetrahedron: $(0,0,0), (1,0,0), (0,2,0), (0,0,3)$.
55. Find the equation of tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at a point $(1,2,3)$ on it.

Short Answer Questions (5 marks questions)

1. Change the origin to point (α, β) and transform the equation $x^2 - 2xy + 3y^2 - 10x + 22y + 30 = 0$. Find (α, β) if the transformed equation does not contain the first degree terms in the new co-ordinates.

2. Shift the origin to a suitable point so that the equation $x^2 - 6x - 4y - 1 = 0$ will be in the form $x^2 = 4by$ and state the value of b.

3. Shift the origin to a suitable point so that the equation $x^2 + 4x - 8y + 12 = 0$ will be in the form $x^2 = 4by$ and state the value of b.

4. Shift the origin to the point $(-1,2)$ and transform the equation $x^2 + y^2 + 2x + 4y = 0$.

5. By rotating the axes, origin being unchanged the expression $ux + vy$ becomes $u'x' + v'y'$, show that $u^2 + v^2 = u'^2 + v'^2$.

6. Reduce the equation to its standard form $x^2 + 2xy + y^2 - 6x - 2y + 4 = 0$.

7. Shift the origin to the centre of the conic and then remove the product term xy of $x^2 + 4xy + y^2 - 2x + 2y = 6$.

8. Find the angle θ through which the axes should be rotated to remove the xy term in $x^2 - 4xy + 4y^2 - 2y + 2 = 0$.

9. Find the angle θ through which the axes should be rotated to remove the xy term in $7x^2 + 12xy - 5y^2 + 4x + 3y = 5$.

10. Find the angle θ through which the axes should be rotated to remove the xy term in $3x^2 - 5xy + 3y^2 = 5$.

11. Remove the product term xy from $4x^2 + 2\sqrt{3}xy + 2y^2 = 7$.

12. If l, m, n are direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$.

13. With usual notation prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

14. Find the angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2.

15. Prove that every equation of the form $ax + by + cz + d = 0$ represents a plane.

16. Prove that every equation of first degree in x, y, z represents a plane.

17. Obtain the equation of a plane in normal form.

18. Find the equation of the plane passing through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z = 5$ and the point $(1, 1, 1)$.

19. Find the equation of the plane passing through the point $(3, 3, 1)$ and perpendicular to the line joining the points $(2, -1, 3)$ and $(4, 2, -1)$.

20. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z = 4$, $2x + y - z + 5 = 0$.

21. Find the equations of the planes bisecting the angles between the planes $x + 2y + 2z = 3$ and $3x + 4y + 12z + 1 = 0$, and specify the one which bisects the acute angle.

22. Find the equation of planes parallel to the plane $x - 2y + 2z = 3$ whose perpendicular distance from the point $(1, 2, 3)$ is 1.

23. Prove that two point $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ lie on the same or different sides of the plane $ax + by + cz + d = 0$, if the expressions $ax_1 + by_1 + cz_1 + d = 0$ and $ax_2 + by_2 + cz_2 + d = 0$ are of the same or different signs.

24. Find the equation of the plane bisecting the angles between the planes $x + 2y + 2z = 9$ and $4x - 3y + 12z + 13 = 0$. Also specify the one which bisects the acute angles.

25. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$.

26. Show that the equation $12x^2 - 2y^2 - 6z^2 - 2xy + 7yz + 6zx = 0$ represents a pair of planes. Also find the angle between them.

27. Show that the equation $x^2 - y^2 + 2z^2 + yz + 3zx + x + y + z = 0$ represents a pair of planes. Also find the angle between them.

28. Find the symmetric form of the equations of the line

$$x + y + z + 1 = 0;\ 4x + y - 2z + 2 = 0.$$

29. Find the equations of a line through $(-2, 3, 4)$ and parallel to the planes $2x + 3y + 4z = 5$ and $3x + 4y + 5z = 6$.

30. Find the angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$, where l, m, n are direction cosines of a line and the plane $ax + by + cz + d = 0$.

31. Find the equations of the line through $(3, 1, 2)$ and perpendicular to the plane $2x - 2y + z + 3 = 0$. Also find the coordinates of the foot of the perpendicular.

32. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$.

33. Find the length of the perpendicular drawn from the point $(5, 4, -1)$ to the line $\frac{x-1}{2} = \frac{y}{9} = \frac{z}{5}$.

34. Show that the line $\frac{x-7}{4} = \frac{y-5}{3} = \frac{z-3}{2}$ intersects the line $5x - 3y + z = 10 : 2x + 7y - 4z = 16$. Also find the coordinates of the point of intersection.

35. Find the equation of the plane containing point $(0, 7, -7)$ and the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$.

36. Show that the lines $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-1}{-1}$ and $\frac{x-4}{2} = \frac{y-2}{-1} = \frac{z-4}{2}$ are coplanar and find the equation of the plane containing them.

37. Show that the lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$ and $\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$ are coplanar and find the equation of the plane containing them.

38. Find the shortest distance and the equations of the line of shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

39. Find the length and equation of the shortest distance between the lines $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ and $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$.

40. Find the length and equation of the shortest distance between the lines $\frac{x-3}{1} = \frac{y\pm4}{1} = \frac{z+1}{1}$ and $\frac{x+6}{2} = \frac{y+5}{4} = \frac{z-1}{-1}$.

41. Find the length and equation of the shortest distance between the lines

$$3x - 9y + 5z = 0 = x + y - z;$$

$$6x + 8y + 3z - 13 = 0 = x + 2y + z - 3.$$

42. Find the length of the perpendicular from the point $(4, -5, 3)$ to the line $\frac{x-5}{3} = \frac{y+2}{-4} = \frac{z-6}{5}$.

43. Find the distance of $(-1, 2, 5)$ from the line through $(3, 4, 5)$ having direction cosines are proportional to $2, -3, 6$.

44. Find the distance of the point $(6, 6, -1)$ from the line $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-1}$. Also find the coordinates of its foot.

45. Prove that the planes $2x - 3y - 7z = 0$, $3x - 14y - 13z = 0$ and $8x - 31y - 33z = 0$ pass through one point.

46. Find the equation of the plane containing the line $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-4}{-2}$ and the point $(0, 6, 0)$.

47. Find the equation of the sphere passing through the points $(3, 0, 2)$, $(-1, 1, 1)$, $(2, -5, 4)$ and having centre on the plane $2x + 3y + 4z = 6$.

48. Find the equation of the sphere passing through the points $(1, 1, 2)$ and $(0, -2, 1)$ and its centre lies on the line $x - 1 = 2 - y = z + 1$.

49. Find the equation of the sphere passing through $(0, 3, 0)$, $(2, 1, -1)$ and whose centre lies on the line $x - y - z = 0 = 2x + 3y$.

50. Find the equation of the sphere passing through the points $(2, 4, -1)$, $(0, -4, 3)$, $(-2, 0, 1)$, and $(6, 0, 9)$.

51. Find the equation of the sphere passing through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$, and $(1, 3, 2)$.

52. Find the equation of the sphere passing through the points $(1, 1, 0)$, $(0, -1, 2)$, $(2, 0, -1)$, and $(2, 2, 0)$.

53. Find the equation of the sphere passing through the points $(1, 0, -1)$, $(2, 1, 0)$, $(1, 1, -1)$, and $(1, 1, 1)$.

54. Find the centre and the radius of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z = 15$.

55. Find the centre and the radius of the circle $x^2 + y^2 + z^2 - 2x - 4y + 2z = 30$, $2x - y + 2z = 7$.

56. Find the equations of the circle which is a section of the sphere $x^2 + y^2 + z^2 + 6y - 6z = 21$ and has its centre at the point $(2, -1, 2)$.

57. Prove that the line $\frac{x+1}{4} = \frac{y-2}{1} = \frac{z-2}{1}$ touches the sphere $x^2 + y^2 + z^2 = 9$. Find the point of contact.

58. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 + 6x - 4y - 6z = 14$, $x + y - z = 0$ and passing through the point $(1, 1, -1)$. Also find centre and radius of this sphere.

59. Show that the spheres $x^2 + y^2 + z^2 - 4x - 2y - 4z + 5 = 0$ and $x^2 + y^2 + z^2 - 6x - 6y + 17 = 0$ touches each other and find their point of touching.

60. Find the points at which the line $\frac{x-7}{2} = \frac{y-6}{1} = \frac{z+5}{-1}$ cuts the sphere $x^2 + y^2 + z^2 - 2x + 3y - 5z = 31$.

61. Find the coordinates of the points where the line $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z-8}{-5}$ intersects the sphere $x^2 + y^2 + z^2 + 2x - 10y = 23$.

62. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9$; $2x + 3y + 4z = 5$ and the point $(1, 2, 3)$.

63. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + 2x - 2y - 2z = 1$; $2x - 2y + z = 1$ and through the point $(3, -1, 1)$.

64. Find the equation of the tangent plane at $P(x_1, y_{1,} z_1)$ to the sphere $x^2 + y^2 + z^2 = a^2$.

65. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$. Also find the point of contact.

Long Answer Questions (10 marks questions)

1. If by rotating the axes through an angle θ, without changing the origin the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is transformed into $a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c' = 0$ then prove that
   i. $a + b = a' + b'$ and
   ii. $ab - h^2 = a'b' - h'^2$.
2. Prove that a general equation of second degree represents a conic.
3. Reduce the equation $5x^2 + 6xy + 5y^2 - 10x - 6y = 3$ to the standard form and name the conic.
4. Reduce the equation $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$ to the standard form and name the conic.
5. Reduce the equation $5x^2 + 6xy + 5y^2 - 4x + 4y = 4$ to the standard form and name the conic.