Anekant Education Society's

Tuljaram Chaturchand College

Department of Mathematics

Class :-Msc 1

Question Bank of Topology :-

2 marks Que.

1) Define Dictionary ordered relation.

2) Define Topology.

3) What is a Discrete Topology?

4) Define

i) Basis for Topology.

ii) Standard Topology.

iii) K-Topology.

iv) Subbasis.

v) Order Topology.

vi) Product Topology.

vii) Subspace Topology.

viii) Hausdorff Space.

ix) Define Homeomorphism.

X) Box Topology

xi) Metric Topology

5) Is the space |Rl connected and justify.

6) Show that |R^n and |R are not Homeomorphic.

7) Define locally connected and locally path connected.

8) Define path component of X.

9) Define Finite intersection property.

10) Find a metric space in which not every closed bounded subspace is compact.

11) State Lebesgue number Lemma.

12) State Uniform continuity theorem.

13) Define limit point compactness.

14) Define sequential compactness.

15) Define local compactness and give an example.

16) State first countability and second countability axioms.

17) Define Lindelof space.

18) Define separable space.

19) Prove that the product of two Lindelof spaces need not be a Lindelof space.

20) Prove that a subspace of Lindelof space need not be a Lindelof.

21) Define normal and regular spaces.

22) Prove that the space |R l is normal.

23) State Urysohn lemma.

24) State Urysohn metrization theorem.

25) State Tietze extension theorem.

26) State Tychnoff theorem.

Short answer que

1) Show that the collection S = { Π_1^{-1}(U)/ U is open in X } ∪{ Π_2^{-1}(v) / v is open in Y} is a subbasis for product topology X x Y.

2) Consider the set Y = [-1, 1] as a subspace of |R. Which of the following sets are open in Y? Which are open in |R?

i) A = {x / ½ < |x|< 1}   ii) B = {x / ½ ≤ |x| ≤ 1}

3) A map f: X → Y is said to be an open map if for every open set U of X the set f(U) is open in Y. Show that Π_1 : X x Y → X and Π_2: XxY → Y are open maps.

4) Let A be subset of topological space X and A′ be the set of all limit points of A then $\overline{A}$ = A ∪ A'.

5) Let A ⊂ X and B ⊂ Y. Show that in the space X x Y $\overline{A \times B} = \overline{A} \times \overline{B}$.

6) Let d and d′ be two metric on set X. Let τ and τ′ be the topologies they induced respectively then prove that τ′ is finer than τ iff for each x ∈ X and $\varepsilon > 0$ ∃ $\delta > 0$ such that $B_{d'}(x, \delta) \subset B_d(x, \varepsilon)$.

7) Show that R × R in the dictionary order topology is metrizable.

8) Let f : S^1 → |R be a continuous map. Show that ∃ a point x of S^1 such that f(x) = f (-x).

Long Answer type

1) Show that every order Topology is Hausdorff.

2) Show that product of two Hausdorff spaces is Hausdorff.

3) Show that subspace of Hausdorff space is Hausdorff.

4) State and prove The Pasting Lemma.

5) Show that the subset (a, b) in |R is Homeomorphic with (0, 1) in |R and the subspace [a,b] is

Homeomorphic with [0, 1] in |R.

6) Let X be a set let τ be a collection of all subsets U of X such that X-U either finite or all of X then show that τ is topology on X & is called finite complement Topology.

7) Let X be a set , τ be a collection of all subsets U of X such that X - U is either Countable or all of X then show that τ is Topology on X.

8) If $\{\tau_\alpha\}$ is a family of topologies of X. Show that intersection is a topology on X.

9) Show that the countable collection β = { (a, b) / a < b and a, b are rationals } is a basis that generates standard topology.

10)If A and B are closed in X and Y , respectively, then prove that A×B is closed in X×Y .

11) Prove that a subspace of a Hausdorff space is a Hausdorff space.

12) Prove that the product of two Hausdorff spaces is a Hausdorff space.

13) Let Y = [0, 1] U {2} be a subset of |R then show that subspace Topology on Y and Ordered Topology on Y are not same.

14) Let X be an ordered set in the order Topology. Let Y be a convex subset of X then show that the order Topology on Y is the same as the subspace topology on Y.

15) Show that projection maps are open maps.

16) Show that every finite point set in Hausdorff space X is closed.

17) Show that every order Topology is Hausdorff.

18) Show that product of two Hausdorff spaces is Hausdorff.

19) Show that subspace of Hausdorff spaces is Hausdorff.

20) Show that X is Hausdorff iff the diagonal is closed in X × X.

21) Prove that f:X→Y is continuous iff f is continuous at every point iff $f(\overline{A}) \subset \overline{f(A)}$ for every subset $A \subset X$ iff $f^{-1}(B)$ is closed for every closed subset $B \subset Y$.

22) Show that the subset (a, b) in |R is Homeomorphic with (0, 1) in |R and the subspace [a, b] is Homeomorphic with [0, 1] in |R.

23) If τ is typology on non-empty set X, then arbitrary ………………. of member of τ belong to τ

a)union b)intersection c) product d) complement

24) Prove that The closure of the product of subsets equals the product of the closures in either Topology.

25) Prove that the Topology on $|R^n$ induced by Eyclidean metric d and the square metric are the same as product Topology on $|R^n$.

26)Prove that The box topology on RJ is finer than the uniform topology which is finer than the product topology.

27) Let f: X→ Y , if the function f is continuous then for every convergent sequence x_n converges to x in X then prove that the sequence $f(x_n)$ converges to f(x) and converse holds if X is metrizable.

28) Prove that the union of collection of connected subspace of X that have point in common is connected.

29)Prove that the image of connected space under a continuous map is connected.

30) Prove that a finite cartesian product of connected spaces is connected.

31) Prove that R^w is not connected in box Topology even though |R is connected.

32) Prove that $|R^w$ is connected in the product Topology.

33) Show that if X is an infinite set , it is connected in the finite complement Topology.

34) Show that if X has discrete Topology then X is totally disconnected.

35) Prove that every path connected space is connected.

36) Give an example of a connected space which is not path connected.

37) Show that no two of the spaces (0, 1), (0, 1], and [0, 1] are Homeomorphic.

38) Prove that a space X is locally connected iff for every open set U of X each component of U is open in X.

39) Let Y be a subspace of X then prove that Y is compact iff every Covering of Y by sets open in X contains a finite sub collection Covering Y.

40) Prove that every compact subspace of a Hausdorff space is closed.

41) Prove that every closed subspace of a compact space is compact.

42) Prove that the image of compact space under continuous map is compact.

43) Let $f : X \rightarrow Y$ be a bijective continuous function. If X is compact and Y is Hausdorff then Prove that f is Homeomorphism.

44) Prove that the Product of finitely many compact spaces is compact.

45) State and Prove The Tube Lemma.

46) Show that if X is compact Hausdorff under both τ and τ' where τ is contained in τ' then either τ and τ' are equal or they are not comparable.

47) Show that a finite union of compact subspaces of X is compact.

48) Show that every compact subspace of a metri space is bounded in that metric and is closed.

49) Let A and B are disjoint compact subspaces of the Hausdorff space. Show that there exists disjoint open sets U and V containing A and B respectively.

50) Show that a subspace A of $|R^n$ is compact iff it is closed and is bounded in the Euclidean metric d or the square metric?

51) Show that for fixed A the function d(x, A) is continuous function of x.

52) State and Prove the Lebesgue number Lemma.

53) State and prove Uniform continuity theorem.

54) Prove that compactness implies limit point compactness but not conversely.

55) Let X be a metrizable space then Prove that the following statements are equivalent

1) X is compact

Ii) X is limit point compact

iii) X is sequentially compact.

56) Show that [0,1] is not limit point compact as a subspace of $|R_l$.

57) Prove that a space X is Homeomorphic to an open subspace of a compact Hausdorff space iff X is locally compact Hausdorff.

58) Prove that a subspace of a first countable is first countable and a countable product of first countable space is first countable.

59) Prove that a subspace of a second countable is second countable and a countable product of second countable space is second countable.

60) Prove that the space |Rl satisfies all the countability axioms but not the second.

61) Show that every metrizable space with a countable dense subset has a countable basis.

62) Prove that a subspace of a regular space is regular and a product of regular spaces is regular.

63) Show that if X is regular every pair of points of X have neighborhoods whose closures are disjoint.

64) Show that every order Topology is regular.

65) Prove that every regular space with countable basis is normal.

66) Prove that every metrizable space is normal.

67) Prove that every compact Hausdorff space is normal.

68) State and Prove Tychnoff theorem.