ANEKANT EDUCATION SOCIETY'S

TULJARAM CHATURCHAND COLLEGE OF ARTS, SCIENCE AND COMMERCE, BARAMATI

AUTONOMOUS

QUESTION BANK

FOR

M.Sc(Sem-II)

STATISTICS

STAT- 4202: Parametric Inference

(With effect from June 2019)

Unit-1:

For 2 Marks:

Q1. Define the following terms with one illustration.

- Sufficient estimator of the parameter by using conditional probability approach.
- Sufficiency.
- Sufficient statistic.
- Joint sufficiency
- Likelihood equivalence.
- Minimal sufficient statistics.
- one parameter exponential family.
- Multi-parameter exponential family.

Q2. Choose the correct alternative of the following:

- Let X_1 , X_2 be a random sample from Poisson distribution with mean λ then $E[(X_1 - X_2)^2]$ is

  a) 2λ b) λ^2 c) λ d) None of these

- Let X_1 , X_2 ,…….., X_n be a random sample from $N(\theta, \theta)$ where θ is unknown, then which of the following statement is not true?

  a) $\left(\sum Xi^2\right)$ is sufficient for θ.

  b) $\left(\sum Xi\right)$ is sufficient for θ.

  c) $\left(\sum Xi, \sum Xi^2\right)$ is jointly sufficient for θ.

  d) Sufficient statistic does not exist

- Which of the following does not belongs to the exponential family of distributions?

  a) $f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}$, $x > 0$ b) $f(x,\theta) = e^{-(x-\theta)}$, $x > \theta$

  c) $f(x,\theta) = \frac{5x^4}{\theta} e^{-x^5/\theta}$, $x > 0$ d) $f(x,\theta) = \frac{e^{-x/\theta} x^2}{2\theta^3}$, $x > 0$

- Let X_1 and X_2 be a random sample from a Poisson (θ). Then number of unbiased estimators of θ is :

  a) infinity b) 3 c) 2 d) 4

- Consider Pitman family of distribution { f(x, θ), θ ∈ Θ} with $f(x,\theta)=\begin{cases}\dfrac{u(x)}{v(\theta)} & if\ a(\theta)<x<b(\theta)\\ 0 & Otherwise\end{cases}$

  where u(x), v(θ) > 0. Suppose a(θ) is increasing and b(θ) is decreasing function of θ. Then minimal sufficient statistic based on random sample of size n is given by:

  a) $\max\{a^{-1}(X_{(1)}),b^{-1}(X_{(n)})\}$
  c) $\max\{a^{-1}(X_{(n)}),b^{-1}(X_{(1)})\}$
  b) $\min\{a^{-1}(X_{(n)}),b^{-1}(X_{(1)})\}$
  d) $\min\{a^{-1}(X_{(1)}),b^{-1}(X_{(n)})\}$

- Two random samples X and Y of sizes n and m respectively from Exp(θ) are likelihood equivalent if and only if
  a) $\sum_{i=1}^{n} X_i \neq \sum_{i=1}^{m} Y_i$
  b) $\sum_{i=1}^{n} X_i > \sum_{i=1}^{m} Y_i$
  c) $\sum_{i=1}^{n} X_i < \sum_{i=1}^{m} Y_i$
  d) $\sum_{i=1}^{n} X_i = \sum_{i=1}^{m} Y_i$

- Let the random variable X follows U (θ, θ+1) then which of the following statement is not correct?
  a) $X_{(1)}$ is sufficient for θ.
  b) $(\bar{X}-\frac{1}{2})$ is an unbiased estimator of θ.
  c) UMVUE will not exist for θ.
  d) Any value of θ in the interval [$X_{(n)}-1$, $X_{(1)}$] is an maximum likelihood estimator.

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For 4 Marks:

1. Let X_1, X_2 are i.i.d. N(θ,1) random variables. Show that (mX_1+nX_2) is sufficient for θ if and only if m=n
2. Define a sufficient statistic and state the Neyman factorization criterion for it. Prove the result indiscrete case.
3. Consider the following p.m.f. P(x=0) $=\theta^{-\alpha}$, P(X=1)$=\alpha\, e^{-\alpha}$ and P(X=2)$=1-e^{-\alpha}$ - $\alpha e^{-\alpha}$.

   Given a random sample of size 2, show that X_1+X_2 is not sufficient for α.
4. Define one parameter exponential family with parameter θ and obtain minimal sufficient statistic for θ.

5. Check whether following distribution belongs to exponential family.Justify your answer.

$$f(x,\theta)=\frac{1}{2}e^{-|x-\theta|} \quad \theta \text{ , x ε R}$$

6. Let $X_1, X_2, \ldots, X_n$ be random sample of size n having following probability density function

$f(x,\theta)=\frac{3\theta^3}{x^4};0<\theta<x<\infty.$ Show that $X_{(1)}$ is minimal sufficient statistic for θ also find its probability density function.

7. Show that $N(\theta,\sigma^2)$ is a member of multi parameter exponential family when both θ and σ^2 are unknown

8. Check whether the following distribution belongs to exponential family.Justify youe answer

$$f(x,\theta)=\frac{1}{\pi[1+(x-\theta)]^2};\theta,x\in R$$

9. Let X_1,X_2,X_3 are i.i.d. Bernoulli(p) and $S_1 = (X_1,X_2,X_3)$, $S_2 = (X_1+X_2,X_3)$, $S_3 = (X_1+X_2+X_3)$ check whether S_1, S_2, S_3 form sufficient partition.

10. Let X1,X2 be i.i.d. U(θ,θ+1), $\theta\in R$. Find minimal sufficient statistic for θ.

11. Show that for a sample of size 3 from Poisson(λ),($X_1, X_2 + X$) is sufficient for λ but not minimal sufficient for λ.

12. If T is a sufficient statistic for θ and $\phi(T)$ is sufficient statistics for θ when ϕ is one to one onto function.

13. Suppose $X_1,X_2,\ldots\ldots,X_n$ is a random sample from beta distribution of first kind with parameters (θ,1).show that T= $\sum\log X_i$ is sufficient statistic for θ.

14. Check whether the following distribution is member of one parameter exponential family.

    i) Ber(θ) and Bin(n, θ), where n is known.
    ii) P(θ) and Geo(θ)
    iii) Discrete Uniform {x = 1, 2, …, N} and continuous U(0, θ).
    iv) Exp(θ) and N(θ, 1)
    v) Cauchy (θ, 1) and X ~ Laplace(θ) with pdf $f(x,\theta)=\frac{1}{2}e^{-|x-\theta|},x>0,\theta>0.$

vi) X ~ Laplace(θ) with pdf $f(x,\theta)=\frac{1}{2\theta}e^{-|x|/\theta}, x>0, \theta>0.$

For 6 Marks:

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta_1,\theta_2)$.Show that ($(\sum X_i, \sum X_i^2)$ form minimal sufficient statistic for $\underline{\theta}$=(θ_1,θ_2).
2. Define m parameter exponential family. Let $X_1, X_2, \ldots, X_n$ be a random sample from m parameter exponential family the obtain a minimalsufficient statistic for the parameter $\underline{\theta}$=($\theta_1,\theta_2,\ldots, \theta_m$)'
3. Define Pitman family and prove the following results
   If a(θ) ↑, b(θ)↓ then min $\{a^{-1}(X_{1}), b^{-1}(X_{n})\}$ is minimal sufficient statistics for θ.
4. Define sufficient partition. Let X_1, X_2 be i.i.d. N(θ,1) then show that $T=lX_1+mX_2$ is sufficient iff l=m.
5. Show that $N(\theta,\sigma^2)$ is a member of multi parameter exponential family when both θ and σ^2 are unknown. Hence obtain a minimal sufficient statistic for (θ,σ^2) based on the random sample of size n drawn from $N(\theta, \sigma^2)$.
6. Define Pitman family and prove the following results
   If a(θ) ↓, b(θ)↑ then Max $\{a^{-1}(X_{1}), b^{-1}(X_{n})\}$ is minimal sufficient statistics for θ.
7. Define likelihood equivalence and explain its usefulness in obtaining a minimal sufficient statistic.
8. Define a sufficient statistic.Give one example of a statistic that is sufficient and one which is not(with justification using the definition only)
9. State and prove Neyman factorization theorem for a parametric family of discrete random variables. Hence check if (i)$X_1 + X_2$ and (ii) $X_1 + 2X_2$ are sufficient where X_1 &X_2 are iid with p.m.f.
   $$f(x,\theta) = (1-\theta)\theta^x, x = 0,1,2,\ldots\ldots,0 < \theta < 1.$$
10. Distinguish between a sufficient statistic and a minimal sufficient statistic. Discuss the relationship between them.

11. State Neyman factorizability Criterion for a statistic T($x_1, \ldots \ldots, x_n$) to be sufficient for the family (L($x_1, \ldots, x_r \theta$),$\underline{x} \epsilon$ S,$\theta \epsilon \Omega$} and using this show that $X_{(n)}$ is a sufficient for θ for a random sample (r.s.) of size n on U(0,θ), $\theta >$ 0.

12. Let {f(x,θ),$\theta \epsilon \Omega$} be a family of probability density functions such that

$$f(x,\theta),=\frac{u(x)}{v(\theta)}, a(\theta) < x < b(\theta)$$

=0, otherwise

State what are the minimal sufficient statistics in each of the following cases.

(i)a(θ) $= a$(constant)

(ii)b(θ) =b(constant)

13. Let $X_{(1)}, X_{(2)}, \ldots., X_{(n)}$ be an ordered sample from{U(0,θ), θ > 0}.Find minimal sufficient statistic for θ.

14. Let X_1, X_2, X_3&X_4 be i.i.d N(θ, 1),θϵR_1.Show that

(i) $(X_1 + X_2)$ is not sufficient

(ii)$(X_3 + X_4)$ is not sufficient

(iii) $(X_1 + X_2), (X_3 + X_4)$ is not sufficient

(iv) $(X_1 + X_2 + X_3 + X_4)$ is sufficient

15. Let $X_1, X_{2,} \ldots., X_n$ be i.i.d. U($\theta - 1, \theta + 1$).Show that $(X_{(1)}, X_{(n)})$ is sufficient for θ but not complete. Is $(X_{(1)}, X_{(n)})$minimal sufficient?

16. Let $X_1, X_{2,} \ldots., X_n$ be random sample from N(θ, 1),then show that T= $X_1+X_2+X_3$ is sufficient statistic for θ.

17. Let $X_1, X_{2,} \ldots., X_n$ be random sample from with density belonging to class {f(x,θ),θ ∈ Ω}which forms an exponential family the prove that joint distribution of $X_1, X_{2,} \ldots., X_n$ is also a member of one parameter exponential family.

18. Let X_1, X_2, …, X_n be a random sample from a Pareto distribution with density function $f(x,\alpha,\beta)=\frac{\beta\alpha^{\beta}}{x^{\beta+1}}, x>\alpha, \alpha>0, \beta>2.$ Find a sufficient statistics when (i) α is known, (ii) β is known and (iii) when both are unknown.

Let X_1, X_2, …, X_n be a random sample from a random variable X with density function $f(x,\theta)=\frac{\theta}{(1+x)^{1+\theta}}, x>0, \theta>0$. Find a minimal sufficient statistic for θ.

19. Let X_1, X_2, …, X_n be a random sample from a random variable X with density function $f(x,\theta)=\frac{\alpha^{\lambda}}{\lambda}e^{-\alpha x}x^{\lambda-1}, x>0, \alpha,\lambda>0$. Find a sufficient statistic when (i) α is known, (ii) λ is known and (iii) when both are unknown.

20. Let X_1, X_2, …, X_n be a random sample from a random variable X with density function $f(x,\theta)=\frac{1}{2}e^{-|x-\theta|}, x>0, \theta>0$. Find a minimal sufficient statistic for θ.

21. Let X_1, X_2, …, X_n be a random sample from a random variable X with pmf $f(x,\theta)=\theta(1-\theta)^{x-1}, x=1,2,..., 0<\theta<1$. Find a minimal sufficient statistic for θ.

22. Let X_1, X_2, …, X_n be a random sample from N(θ, θ^2) distribution. Show that $(\bar{X},S^2)$ is Minimal Sufficient Statistic.

23. Let X_1, X_2, …, X_n be a random sample from a random variable X with density function $f(x,\theta)=\frac{\theta}{(1+x)^{1+\theta}}, x>0, \theta>0$. Find a minimal sufficient statistic for θ.

24. Check whether following distribution belongs to two parameter exponential family.

$$f(x,y)=\binom{x}{y}p^{y}(1-p)^{x-y}e^{-\theta}\frac{\theta^{x}}{x!};\ y=0,1,...x; x=0,1,...; 0<p<1; \theta>0$$

25. Check whether following distribution belongs to two parameter exponential family.

$$f(x,\theta,\lambda)=\begin{cases}\frac{1}{\lambda\theta^{\lambda}}x^{\lambda-1}e^{\frac{-x}{\theta}} & x>0,\theta,\lambda>0\\ 0 & otherwise\end{cases}$$

26. Let X1, X2, …, Xn be a random sample of size n from U(θ, θ+1) distribution. Obtain minimal sufficient statistic for θ.

27. Check whether the following distribution is member of one parameter exponential family.

X ~ Laplace(θ) with pdf $f(x,\theta)=\frac{1}{2}e^{-|x-\theta|}, x>0, \theta>0.$

X ~ Laplace(θ) with pdf $f(x,\theta)=\frac{1}{2\theta}e^{-|x|/\theta}, x>0, \theta>0.$

Unit-2:

For 2 marks:

Q1. Define the following terms:

- Fisher information function.
- Fisher information matrix.
- Unbiased estimator.
- Estimable function.
- Minimum variance unbiased estimator
- Minimum variance bound unbiased estimator
- Ancillary statistic
- Complete family.
- Complete sufficient statistic.
- Crammer Rao inequality.

Q2. Choose the correct alternatives of the following:

- Consider Cramer family of distribution $\{f(x, \theta), \theta \in \Theta\}$. Let T be unbiased for $\psi(\theta)$. Which of the following is Cramer-Rao inequality?

  a) $P(|T-\theta|>k)\le \frac{Var(T)}{k^2}$

  b) $Var(T)\le \frac{[\psi'(\theta)]^2}{I_X(\theta)}$

  c) $P(Sup|T-\psi(\theta)|>k)\le \frac{Var(T)}{k^2}$

  d) $Var(T)\ge \frac{[\psi'(\theta)]^2}{I_X(\theta)}$

- Crammer Rao inequality is regarding

  a) Probability outside the critical region

  b) Variance of unbiased estimator

  c) Bound on power of UMP test

  d) Probability of union of two random events.

- Let X_1 and X_2 be a random sample from Ber(θ). Then which of the following is not estimable?

a) $\theta(1-\theta^2)$ b) θ^2 c) θ d) $1-\theta^2$

- Which of the following statement is not true?
  a) Minimum Variance Unbiased Estimator (MVUE) is unique.
  b) Minimum Variance Bound Unbiased Estimator (MVBUE) always exist.
  c) Let $X_1, ..., X_n$ be a random sample from $N(\theta, 1)$. Then $\sum x_i$ is sufficient of θ.
  d) Likelihood equivalence leads to minimal sufficient statistics.
- Let X be a random variable with probability density function
  $f(x, \theta) = e^{-(x-\theta)}$, $x \geq \theta$, $\theta \in (-\infty, \infty)$. then MLE of θ is
  a) $\frac{\sum_{i=1}^{n} X_i}{n}$ b) $\frac{\sum_{i=1}^{n} X_i}{n} - 1$ c) $X_{(n)}$ d) $X_{(1)}$
- Let $f(x, \theta)$ be the probability density function of a random variable X for which differentiation under integration sign is permissible then,
  $E\left(\frac{\partial}{\partial\theta}\log f(x,\theta)\right)$ is equal
  a) 0 b) $I(\theta)$ c) 1 d) Var(X)
- Let X and Y be i.i.d. Poisson random variable with mean $\frac{\theta}{2}$ then
  a) $T_1 = (X - Y)^3$ is unbiased estimator of θ
  b) $T_2 = (X - Y)^2$ is unbiased estimator of θ.
  c) $T_3 = (X - Y)$ is unbiased estimator of θ.
  d) Unbiased estimator of θ does not exist.

For 4 Marks:

1. Show that unbiased estimator T of $\psi(\theta)$ iff T is uncorrelated with every unbiased estimator of zero.
2. Let X_1, X_2 be a random sample of size two from $P(\theta)$.Is $\psi(\theta) = \theta^2$ estimable?.Justify.

3. Define Complete sufficient statistic. Show that for Poisson distribution with parameter θ, T=$\sum X_i$ is complete statistic for θ.
4. Define MVBUE. Under regularity conditions show that if T is MVBUE of θ then it is sufficient for θ
5. Let $X_1, X_2, \ldots, X_n$ be a random sample from P(λ) ,λ>0 and n>1.Find an unbiased estimator of λ^2 based on this unbiased estimator obtain uniformly minimum variance unbiased estimated for λ^2.
6. Obtain information function of geometric distribution having parameter p.
7. Show that for a random sample of size n from Poisson with mean $\lambda, \lambda > 0, T = \sum_{i=1}^{n} X_i$ is complete for λ.
8. Let $X_1, \ldots., X_n$ be i.i.d .U(0,θ) and let T=Max $(X_1, \ldots., X_n)$,n ≥2.Find unbiased estimator of $\psi(\theta) = 1/\theta$ based on T.
9. Define complete statistic. Show that for Poisson distribution with parameterθ, $T = \sum_{i=1}^{n} X_i$ is complete for θ.
10. Let X1, X2 be a random sample from Bernoulli (p) then show that ψ(p) = p3 is not estimable.

For 6 Marks:

1. State and prove necessary and sufficient condition for existence of MVUE.
2. State and prove necessary and sufficient condition for existence of MVBUE.
3. Let $(X_1, X_2, \ldots., X_n)$ be i.i.d b(1,θ).Show that $T_1(X_1, X_2)$=1 if X_1=1,and $X_2 = 1$ and zero otherwise is an unbiased estimator of θ^2 and obtain Rao-Blackwellized version of T_1 w.r.t. T=$\sum_{i=1}^{n} X_i$ which is known to be sufficient for θ.
4. State and prove Rao Blackwell theorem.
5. State and prove Lehman scheffe theorem
6. State and prove Basu's theorem.
7. State and prove Crammer Rao inequality.
8. Let X have N(0, σ2) distribution. Show that X is not complete but X2 is complete.
9. Let X ~ Bin(n, p), where n is known. Find MVUE of p2 and p(1-p).

10. Let X1, X2, …, Xn be a random sample from P(θ) distribution. Find MVUE for P[X ≤ 1].

11. Let X1, X2, …, Xn be a random sample from a random variable X with pmf $f(x,\theta)=\theta(1-\theta)^{x-1}, x=1,2,..., 0<\theta<1.$ Find MVUE of θ.

12. Define complete family of distribution. Show that {N(θ, 1), θ ∈ (-∞, ∞)} is a complete family of distribution.

13. Let X have N(0, σ2) distribution. Show that X is not complete but X2 is complete.

14. Let X1, X2, …, Xn be a random sample from a random variable X with pmf $f(x,\theta)=\theta(1-\theta)^{x-1}, x=1,2,..., 0<\theta<1.$ Find MVUE of θ.

15. Let X1, X2, …, Xn be a random sample from Poisson (θ) distribution. Suppose Ψ(θ) = e-θ is the parametric function of interest. Then show that, Ψ(θ) is an estimable function but MVBUE of Ψ(θ) does not exist

Suppose $T_1=\begin{cases}1 & ;if\ X_i=0\\ 0 & ;if\ X_i\neq 0\end{cases}$. Let M= ∑ Xi then carry out Rao Blackwellisation of T1 with respect to M.

16. Define Fisher information in a Sample Ix (θ) and information in a statistic IT (θ). Show that IT (θ) ≤ Ix (θ).

17. Let $X_1, X_2, \ldots, X_n$ be a random sample from N(θ, 1) and $\psi(\theta) = \theta^2$. Give an unbiased estimator of ψ(θ). Examine whether its variance attains the Crammer–Rao Lower Bound.

18. Let X1, X2, …, Xn be a random sample of size n ≥ 3 from Bernoulli (p). Obtain an unbiased estimator of parameter p2(1 - p). Hence, find UMVUE of $p^2(1 - p)$.

19. Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with density function $f(x,\theta)=\frac{1}{\theta}e^{-\theta x}, x>0, \theta>0.$ Show that $Y=\sum_{i=1}^{n}x_i$ is Complete.

20. Show that the one parameter exponential family of distribution is a Complete family of distribution.

21. Define complete statistic. If $\{f(x, \theta), \theta \in (-\infty,\infty)\}$ is one parameter exponential family, then show that $T = \sum_{i=1}^{n} K(x_i)$ is complete for θ.

22. Let $(X_1, X_2, \ldots, X_n)$ be i.i.d b(1,θ).Show that $T_1(X_1, X_2)$=1 if X_1=1,and $X_2 = 1$ and zero otherwise is an unbiased estimator of θ^2 and obtain Rao- Blackwellized version of T_1 w.r.t. T=$\sum_{i=1}^{n} X_i$ which is known to be sufficient for θ.

23.Let $X_1, X_2, \ldots, X_n$ be i.i.d. U$(\theta - 1, \theta + 1)$.Show that $(X_{(1)}, X_{(n)})$ is sufficient for θ but not complete. Is $(X_{(1)}, X_{(n)})$minimal sufficient?

Unit-3:

For 2 Marks:

Q1. Define the following terms with one illustration.

- Critical region
- Test function
- OC curve
- Level of significance
- Size of the test
- MP test
- UMP test
- MLR property
- UMPU test
- Type I year
- Type II error
- Composite hypothesis

Q2. Choose the correct alternatives of the following:

- Let $X_1, X_2, \ldots, X_n$ be a random sample from N(θ, 1) then SELCI of level α is:

  a) $\left(\overline{X} - \frac{Z_{1-\alpha/2}}{\sqrt{n}}, \overline{X} + \frac{Z_{1-\alpha/2}}{\sqrt{n}}\right)$ c) $\left(\overline{X} - \frac{Z_{1-\alpha/2}}{\sqrt{n}}, \overline{X} + \frac{Z_{\alpha/2}}{\sqrt{n}}\right)$

b) $\left(\overline{X}-\frac{Z_{\alpha/2}}{\sqrt{n}},\overline{X}+\frac{Z_{\alpha/2}}{\sqrt{n}}\right)$ d) $\left(\overline{X}-\frac{Z_{\alpha/2}}{\sqrt{n}},\overline{X}+\frac{Z_{1-\alpha/2}}{\sqrt{n}}\right)$

- Let X~N(θ, 4) distribution then 95% confidence interval for θ is

  a) (X-2, X+2) b) (X-0.75, X+0.75)

  c)(X-3.92, X+3.92) d) (X-4, X+4)

For 4 Marks:

1. Define MLR property and show that distributions belonging to the one parameter exponential family possess this property.
2. State the Neymann-Pearson lemma for testing a simple hypothesis against a simple alternative .Prove the sufficiency part for 0< α <1.
3. Define a test function. Distinguish between a randomized and a non-randomized test and explain the advantage in using the former.
4. Find size and power of these test. Which one will you prefer? Why?
5. Define UMP and UMPU test.
6. Find UMP test α test for testing $H_0: \theta' = \theta_0$Vs $H_1: \theta < \theta_0$,when a random sample is taken from $\{U(0,\theta), \theta > 0\}$ distribution.
7. Let $f(x,\theta)$=$\theta e^{-x} + (1-\theta)xe^{-x} x > 0, 0 < \theta < 1$.On the basis of a sample of size one, obtain MP test of size $\alpha = e^{-1}$, to test $H_0 = 1$ vs $H_1: \theta = 1/2$.
8. Explain the terms: Rejection region, Errors of two types. Why the probabilities of two types of errors cannot be minimized simultaneously?
9. Let $X_1, \dots, X_n$ be i.i.d exponential random variables with mean θ.Obtain the most powerful test for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$(<θ_0).
10. Consider a sample of size 1.Obtain a most powerful size α test for $H_0: f_0(x) \sim N(0,1)$ against $H_1: f_1(x) \sim cauchy(0,1)$. Also obtain its power.
11. If X is a random variable having pdf $f(x,\theta) = \frac{1}{2}e^{-|x-\theta|}; x\epsilon R, \theta\epsilon R$, show

that the family has MLR property in x.

12. Give an example (with brief justification) in which a UMP test exists against a two sided alternative

13. Obtain MP level αtest for testing $H_0: \theta = \theta_0$Vs $H_1: \theta = \theta_1 (\theta_1 > \theta_0)$, based on a random sample of size n from exponential distribution with mean θ.

14. Show that there does not exist UMP test for testing $H_0: \theta = 0$ against $H_0: \theta \neq 0$ at level α,when $(x_1, \dots \dots, x_n)$ is r.s. from N$(\theta, 1)$.

15. Show that the same test as given in above question continues to be UMP level α test for testing $H_0: \theta \leq 1$vs $H_1: \theta > 1$.

16.

For 6 Marks:

1. State Neyman-Person Lemma part-A and Part-B and prove Part-A.
2. Let X be a discrete r.v. with pmf given by $f_0(x) = .05\ for\ x = 1,2, \dots .,20\ under\ H_0\ and\ under\ H_1.$

   $f_1(x)$=.06 for x=1,

   $=.15$for x=2.3

   $= \frac{.10}{17}$ for x=4,5,….,20
3. Define $\phi_1(x) = 1,2$ and zero otherwise and $\phi_2(x)$=1/2 if x=2,3

   =1 if x=1

   =0 otherwise.
4. Show that both $\phi_1\ and\ \phi_2$ are MP test of level $\alpha = 10$ with same power. Do ϕ_1, ϕ_2satisfy NP lemma?
5. For testing a composite null hypotheses $H_0: \theta \epsilon \Omega_{H_0}$vs $H_1: \theta \epsilon \Omega_{H_1}$ define (i) size function,(ii)power function(iii) level α test and (iv) UMP level α test when H_1 is also composite.
6. For testing $H_0: \theta = 1$ vs $H_1: \theta > 1$ on the basis of a sample of size n on $f(x, \theta)$=$\theta e^{-\theta x}, x > 0, \theta > 0$.Obtain UMP level α test and show that its power function is monotone.
7. Define a UMP test. Obtain such test for a U(0,θ)r.v. based on a random sample of size n, for testing $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$. How is the test modified if alternative is left sided?

8. Sketch the power curve of the UMP test for $H_0: \theta = \theta_0$against $H_1: \theta > \theta_0$.for a normal variate with mean θ and variance 1.Give justification briefly.
9. Define MLR property and illustrate with example as well as counter example. How is it useful in deriving optimal tests?
10. Show that a UMP test does not exist for a two sided alternative, to test a simple hypothesis about the population mean of a normal r.v. with known variance.
11. Let Let $X_1, \dots, X_n$ be independent and identically distributed random variables with p.d.f. $f(x, \theta), \theta \epsilon \Omega$.Consider the problem of testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.Does a UMP test exist for the above problem? If yes, give an example of a distribution for which it exists and derive the UMP test for the same.
12. State the Neymann-Pearson fundamental lemma for test functions.
13. Let $X_1, \dots, X_n$ be i.i.d random variables from $f(x, \theta), \theta \epsilon \Omega = \{\theta_0, \theta_1\}$.Let $H_0: \theta = \theta_0$ and $H_1: \theta = \theta_1$. Show that for every $\alpha, 0 < \alpha < 1$,there exists a k≥0 and the test $\phi_k(\underline{x})$, for which E[$\phi_k(\underline{x})$] $= \alpha$ where

$$\phi_k(\underline{x}) = \begin{cases} 1 if\ L_1(\underline{x}) > kL_0(\underline{x}) \\ \gamma\ if L_1(\underline{x}) = kL_0(\underline{x}) \\ 0 \qquad otherwise \end{cases}$$

14. Let $f(x, \theta)$=1/π 1/1+(x-$\theta)^2$,xϵR_1 and suppose it is desired to test $H_0: \theta = 0$ vs $H_1: \theta = 1$ on the basis of a single observation. Show that the test $\psi(x) = 1$ if 1<x<3 and zero otherwise is the MP test of its size.
15. Let $f(x, \theta)$=$\theta x^{\theta-1} 0 < x < 1$ and $\theta > 0$.Let $(X_1, \dots, X_n)$ be a random sample of size n on $\{f(x, \theta), \theta > 0\}$.Obtain UMP level α test for testing $H_0: \theta \leq 1$ Vs $H_1: \theta > 1$.
16. Define monotone likelihood ratio property and check whether $f(x, \theta) = \frac{1}{2}exp\{-|x - \theta|\}, x \epsilon R, \theta \epsilon R_1$ has this property.
17. how that UMP level α test does not exist for testing $H_0: \theta = \theta_0$Vs $H_1: \theta \neq \theta_0$, based on a random sample of size n from {N(θ, 1), $\theta \epsilon \mathcal{R}$}.Suggest UMP unbiased test for the same problem.
18. Let $\{X_i\}_i^n$ be i.i.d with distribution $f(x, \theta), \theta \epsilon \Omega$.Suppose it is desired to test $H_0: \theta = \theta_0$ vs $H_1: \theta = \theta_1$.Show that any test $\varphi_k(x\sim), k \geq 0$ of the form.

$$\varphi_k(x) \begin{cases} = 1 \quad if\ L(x\sim\theta_1) > kL(x\sim,\theta_0) \\ = \gamma\ \ if L(x\sim\theta_1) = kL(x\sim,\theta_0) \\ = 0\ if L(x\sim\theta_1) < kL(x\sim,\theta_0 \end{cases}$$

Is an MP test of size E [φ_k(x) | θ_0].

19. What is MLR property ? Using MLR property obtain UMP test for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta > \theta_0$, using a random sample of size n from $f(x,\theta) = e^{-(x-\theta)}, x > \theta$
20. Let X follows the p.d.f f(x) =$\theta x^{\theta-1}$, 0<x<1, θ >0.Derive the UMP test for H_0: $\theta = 1$ against H_1:$\theta < 1$ based on a sample of size n.
21. Derive a most powerful test based on a single observation on a r.v. x for H_0:X follows Weibull distribution with p.d.f. x exp $-x^2/2$ against H_1:X follows folded normal distribution with p.d.f.($\sqrt{2/\pi}$)exp $-x^2/2$.Sketch the two density functions. Explain why the critical region obtained above is intuitively reasonable.
22. Define a UMP test. Explain how MLR property is useful in deriving such a test. Show that a MP test is always unbiased. Hence sketch the power function for testing H_0: mean of a normal distribution with unit variance, is zero, against a one sided alternative .Explain briefly why a UMP test does not exist against a two-sided alternative.
23. Let X be a discrete random variable with pmf under H_1&H_0 given by

X=x	1	2	3	4
$P_0(x)$	0.45	0.05	0.05	0.45
$P_1(x)$	0.20	0.30	0.30	0.20

Define $\phi_1(x) = \begin{cases} 1\ \ if\ x = 2 \\ 0\ if\ x = 1,3,4 \end{cases}$ & $\phi_2(x) = \begin{cases} 1\ \ if\ x = 3 \\ 0\ \ if\ x = 1,2,4 \end{cases}$

Unit-4:

For 2 Marks:

Q1. Define the following terms with one illustration.

- Confidence Interval.
- Shortest Expected Length C.I.
- Uniformly Most Accurate C.I.
- Prior Distribution.
- Posterior distribution.
- Loss Function.
- Conjugate Family.
- Coefficient of Confidence Interval.
- Equal tailed Confidence Interval.
- Pivotal quantity
- Baye's estimator.
- Minimax Decision Rule
- Risk function.

Q2. Choose the correct alternatives of the following.

- If ($X_{(1)}$,$X_{(n)}$) is a confidence interval for population median then the confidence coefficient is....

  a) $1-\frac{1}{2^n}$ b) $1-\frac{1}{2^{n-1}}$ c) $\frac{1}{2^n}$ d) $1-\frac{1}{2^{n+1}}$
- Pivotal quantity used for the construction of confidence interval for σ^2, in case of $N(\mu, \sigma^2)$ distribution follows.....

  a) Normal distribution b) Chi square distribution
  c) t- distribution d) F -distribution
- We prefer the confidence interval with confidence coefficient $(1-\alpha)$if it has....

  a) Shortest width b) equidistant confidence limits from parameter
  c)longest width d) one sided confidence limits
-

For 4 Marks:

1. Explain the concept of a confidence interval (CI).What is a pivotal quantity? Show with example its use in obtaining a CI.

2. Explain the connection between CI and test of hypothesis. What is a uniformly most accurate confidence bound? How can we get such a bound for the mean of a normal distribution with known variance?
3. Suppose a random sample of size 15 from a Bernoulli distribution with parameter p is as follows:
   1,0,0,1,1,1,0,0,0,0,1,0,1,0,0
   The prior distribution of p is a Beta distribution with parameters α=2 and β=
   Using squared error loss function. Obtain Baye's estimate of p.
4. The diameter of 10 ball bearings were measured in suitable units are as follows:
   12.01,12,12.02,12.01,12.02,12.01,12.03,12.02,12.01,12.00.
   Find the 95% C.I. for mean diameter assuming the diameter to be normally distributed.
5. If X1,X2,…,Xn is a r.s. from exponential with mean $1/\theta$ find the $(1-\alpha)100\%$ C.I. for θ.

For 6 Marks:

1. Obtain Shortest Expected Length Confidence Interval (SELCI) of level $(1-\alpha)$ for θ based on independent random sample of size n from $N(\theta, 1)$ by using pivotal quantity depending on minimal sufficient statistic for θ.

2. Explain the term Posterior distribution. If X is a random variable having Ber(θ) distribution and $\theta \sim U(0, 1)$ then obtain posterior distribution of θ.
3. Define uniformly most accurate (UMA) confidence interval. Let X1, X2, ... , Xn be a random sample from $N(\theta, \sigma 2)$ where $\sigma 2$ is unknown derive equal tailed confidence interval for θ of level $(1-\alpha)$.

4. Define minimax decision rule. Let X~ Ber (p), $p \in \{\frac{1}{4}, \frac{1}{2}\}$ and A= $\{a_1, a_2\}$. Let the loss function given by

	a_1	a_2
$P_1 = \frac{1}{4}$	1	4

$P_2 = \frac{1}{2}$	3	2

Obtain minimax decision rule if the four decision rules are

i) $\delta_1(0) = \delta_1(1) = a_1$ ii) $\delta_2(0) = a_1$, $\delta_2(1) = a_2$

iii) $\delta_3(0) = a_2$, $\delta_3(1) = a_1$ iv) $\delta_4(0) = \delta_4(1) = a_2$

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f(x,\theta) = \theta x^{\theta-1} \quad 0 < x < 1,\ \theta > 0$$
$$0 \qquad otherwise$$

Obtain UMA confidence interval for θ with confidence coefficient $(1-\alpha)$.

6. Describe a method of obtaining a confidence interval for a parameter θ based on a large sample.Hence obtain $100(1-\alpha)$ % confidence interval for θ the mean of an exponential distribution.
7. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population with p.d.f.

$$f(x,\theta) = \theta e^{-\theta x} \quad ;\ x>0$$
$$= 0 \quad ;\ otherwise$$

Let the parameter θ have the p.d.f.

$$h(\theta) = e^{-\theta} \quad ;\ \theta > 0$$
$$= 0 \quad ;\ otherwise$$

Obtain Bayes solution for θ using a squared error loss function and $Y = \sum Xi$.

8. If X follows a Binomial distribution with parameters (k,p) and the prior distribution of p is Beta distribution of first kind with parameters (α,β),then find the posterior distribution of p based on a random sample $X_1, X_2, \ldots, X$ of size n from the Binomial distribution.
9. If X follows a Poisson distribution with parameters λ and the prior distribution of λ is Gamma distribution with parameters (α,β),then find the posterior distribution of λ based on $Y = \sum Xi$ .where $X_1, X_2, \ldots, X_n$ is r.s. of size n from the Poisson distribution.
10. Let $X_1, X_2, \ldots, X_n$ be a r.s. of size n from a Bernoulli distribution with parameters p as the probability of success.Let $Y = \sum Xi$. and the prior distribution of p is Beta distribution of first kind with parameters (α,β).Obtain Bayes estimator of p using a squared error loss function .

11. Let $X_1,X_2,\ldots,X_n$ be a r.s. of size n from a Binomial distribution with parameters k and p .Let $Y=\sum Xi$. and the prior distribution of p is Beta distribution of first kind with parameters (α,β).Obtain Bayes estimator of p using a squared error loss function .
12. Let $X_1,X_2,\ldots,X_n$ be a r.s. of size n from a Normal distribution with parameters μand σ^2 . where σ^2 is known. Let $Y=\overline{X}$ where $\overline{X}$ is sample mean .Let the prior distribution of μ be normal with mean μ_0 and standard deviation σ_0 .Obtain Bayes estimator of μ using a squared error loss function .

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