Department of Mathematics

F.Y.B.Sc.(Computer Science)

Question Bank

Paper–I:*Discrete Mathematics*

Answer in One Sentence(or in 2 – 3 lines) **(2 marks questions)**

1. Find contrapositive of "If study then I pass".
2. Translate the following in to symbolic form
   a.Some Horses run faster than some cars.
   b.All integers are rationals.
3. Justify whether true or false.$\sim(p \to q) \equiv \sim p \to \sim q$
4. What rules of inference is used in the following argument?

   "Alice is mathematics major.Therefore Alice is either mathematics major or a computer science major."

5. Prove the validity of the following argument by using method of indirect proof.
   $$p \to q, r \to \sim t, p \vee r, t \vdash q$$
6. Prove the following equivalence using truth table.
   $$p \to q \equiv \sim q \to \sim p$$
7. Write negation of each of the following.
   a. $\exists x(P(x) \wedge \sim Q(x))$
   b. $\forall x(P(x) \wedge Q(x))$
8. Draw a Hasse diagram for the relation 'divides'on set A={1,3,4,8,12,24}.
9. Find a complement of the elements c and b in the following lattice.
10. Give an example of a lattice which is not distributive but complemented.Justify.
11. State idempotent and absorption laws.
12. Find the number of permutation that can be formed from all the letters of the word "UNUSUAL".
13. How many arrangements are there of letters in the word 'SWINE'?
14. In how many ways can 4 identical red pens,6 identical green pens, 3 identical blue pens be arranged ?
15. In how many ways one right and one left shoe be selected from six pairs of shoes without obtaining a pair .
16. How many non-negative integer solutions are there in the equation $x + y + z + w =$ 10?.

17. Find First six terms of the sequence defined by the following recurrence relation.

$$a_n = a_{n-1} + 3a_{n-2\,with}\; a_0 = 1, a_2 = 2$$

18. Write recurrence relation for the sequence 2,5,8,11.
19. Solve the recurrence relation
    $a_r - 2a_{r-1} = 0.$
20. Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$
21. Justify true or false.The following is a homogeneous linear recurrence relation

$$a_{n+3} - 5a_{n+4} + 6a_{n+3} * a_{n+2} - 4a_n = 0$$

22. Solve the recurrence relation $an - a_{n-2} = 0$
23. Solve the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = 0$
24. Find particular solution of the recurrence relation $a_n - 4a_{n-2} = 3n$
25. If characteristic roots of recurrence relation are 2,2,3 with $f(r) = (r^2 + 1)2^r$.
26. Find particular solution of the recurrence relation .
    $a_n - 4a_{n-2} = 3n.$
27. Give an example of a relation on the set A={1,2,3} which is not symmetric but transitive .Justify your answer.
28. List all partitions of a set A={a,b,c}.
29. Define partial order relation. Give an example of partial order relation.
30. Let A={a,b,c,d}. Determine whether following relation R on A in transitive or not. Where R={(a,a) , (a,b) , (b,c)}.Justify.
31. Let A be any non- empty set. Is every relation on A an equivalence relation? Justify.
32. Let A={1,2} and B={a,b}.List all possible functions from A to B.
33. Draw diagraph of the relation R given by aRbiff$a + b \leq 5\;, a, b \in A$ Where A={1,2,3,4,8}.
34. Define a relation R on Z by xRyiff$(x - y) \in Z$.Show that R is an equivalence relation .Determine the equivalence classes of 3 and $\sqrt{2}$.
35. Let A={1,2,3,4} Write an equivalence relation on A and write matrix of relation.

Short Answer Questions **(4 marks questions)**

1. Let $p \equiv Food\ is\ good$

$$q \equiv The\ service\ is\ good$$

$$r \equiv The\ rating\ is\ 3\ star$$

   Convert the following statement in to symbolic form.

   a. Either the food is good or service is good but not both.
   b. Either the food is good or service is good or both.
   c. It is not the case that both food is good and rating is 3 star.
   d. If both food and service are good then rating will be 3 star.

2. John either always tells the truth or he always lies.He made statements.

   a. I love Lucky.
   b. If I love Lucky then I also love Vivian.
      Determine whether John really love Lucky.

3. Test for tautology.

$$[(p \wedge q) \rightarrow r] \rightarrow [p \rightarrow (q \rightarrow r)]$$

4. Determine if the following properties are tautology .Justify your answer.

   a. $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$
   b. $((p \vee q) \wedge \sim p) \rightarrow q$

5. Test the validity of an argument by truth table .

$$a \rightarrow \sim b, b, \sim c \rightarrow a \vdash c$$

6. Prove that $[q \rightarrow (b \wedge c)] \equiv [(a \rightarrow b) \wedge (a \rightarrow c)]$.

7. State and prove Demorgans law.

8. Simplify using laws of logic.

$$[p \wedge (\sim p \vee q)] \vee [(\sim p \wedge q) \vee \sim q]$$

9. Test validity of argument without using truth table.

$$p \vee q, p \rightarrow q, \sim r \vdash q$$

10. Draw a Hasse diagram of the poset$D48$,the set of all divisors of 48 with respect to partial order relation “divides”.

11. Suppose that a function is given by a Boolean expression such that

$$f(x1, x2, x3) = (x1 \vee x2) \wedge x3$$

12. Prove that in a complemented distributive lattice ,a complement of an element is unique.

13. Find the conjunctive normal form of a Boolean expression.

$$E(x, y) = (x \vee \bar{y}) \wedge (x \wedge \bar{y})$$

14. Consider the set of alphabet $S = \{a, b, c, d, e, f, g, h, i, j, k\}$.Determine the number of words of length 6 formed using distinct element of S which contains pattern ab or ba.

15. A committee of 5 is to be selected among 6 boys and 5 girls .Determine the no. of ways of selecting the committee , if it is to consist of atleast one boy and one girl.

16. How many numbers are there between 100 and 1000 in which all the digits are distinct?

17. How many integers from 1 to 567 are divisible by 3 or 5 or 7?

18. Determine the number of integer from 1 to 250 that divisible by any of the integers 2,3,5,7.

19. How many integers between 999 and 10000 either begin or end with 3?

20. Consider the recurrence relation
$a_n = a_{n-1+}2a_{n-2}$; with$a_9 = 3$ and $a_{10} = 5$. Find a_7 *and* a_{12}.

21. Solve the recurrence relation :

$a_{n=}3a_{n-1} + 4a_{n-2} : n \geq 2anda_0 = a_1 = 1$.

22. Solve the recurrence relation :

$a_r - 7a_{r-1} + 10a_{r-2} = 0$with$a_0 = 10, a_1 = 41$

23. Write general form of the solution of a linear homogenous recurrence relation with constant coefficient if its characteristic equation has roots -1,-1,-1,7,7.

24. Solve the recurrence relation :

$b_n = 3b_{n-1} - 2b_{n-2}$with initial condition$b_1 = 5, b_2 = 3$.

25. Solve the recurrence relation for Fibonacci sequence with initial condition $a_0 = 0, a_1 = 1$.

26. Solve the recurrence relation:$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 3 \; with \; a_0 = \frac{1}{9}, a_1 = \frac{1}{9}, a_2 =2$.

27. Solve $a_n - a_{n-2} = n - 2$.

28. Solve $a_r - 3a_{r-1} - 4a_{r-2} = 4r \; r \geq 2$.

29. Solve $a_r = 3a_{r-1} + r^2 - 3 \; with \; a_0 = 1$

30. Let A={a,b,c};then write equivalence relation on set A in matrix form and draw digraph of $[M(R)]^2$.

31. Prove that any two equivalence classes are either disjoint or identical.

32. Let R be the relation on the set of ordered pairs of positive integers defined as (a,b)R(c,d) iffa+d=b+c. Determine whether R is an equivalence relation.

33. Draw digraph of the relation R given by aRb if a+b is even number ,$a, b \in A$ where A={1,2,3,4,5}.Also find M(R).

33. Consider the set X={1,2,3,4} and R be a relation defined on X as R={(1,3) , (1,4) , (2,1) , (2,3) , (3,2) , (3,4) , (4,3)}.Obtain transitive closure of R using Warshall's algorithm.

34. Let R be a relation on Z , the set of integers , defined as xRy if and only if 5x+8y is divisible by 13.Show that R is an equivalence relation.

Long Answer Questions **(8 marks questions)**

1. Let a_n be the recursive relation defined by
   $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, $n \geq 3$,with initial conditions $a_0 = a_1 = a_2 = 1$
   a. Obtain a_3 , $a_4, a_5, a_{6.}$
   b. Prove that $a_n \leq 2^{n-1}$ $\forall\, n \geq 1$.
2. Solve the following recurrence relation :$a_n = a_{n-1} + 2a_{n-2} + n$, $a_0 = 0$, $a_1 = 1$
3. Solve the recurrence relation : $a_r - 9a_{r-1} + 20a_{r-2} = 2r$
4. Solve the recurrence relation :$a_r + 3a_{r-1} + 3a_{r-2} + a_{r-3} = r$ where $a_0 = -1$, $a_1 = 2$, $a_2 = 3$
5. Solve the recurrence relation : $a_r = a_{r-1} + 2a_{r-2} + 4(3^r)\ with\ a_0 = 11, a_1 = 28$
6. Solve the following recurrence relation
   $a_r = a_{r-1} + 2a_{r-2} + 7(5^r)\ with\ initial\ condition\ a_0 = 11$, $a_1 = 28$

8.Solve the recurrence relation: $a_n - 9a_{n-1} + 20a_{n-2} = 2 * 5^n\ with\ a_0 = 0$, $a_1 = 1$.

9. Solve the recurrence relation :$a_r = 3a_{r-1} + 3a_{r-2} + a_{r-3} = r\ where\ a_0 = -1$, $a_1 = 2$, $a_2 = 3$.

10. Solve the recurrence relation :$a_n - 2a_{n-1} = 3^n\ where\ a_1 = 1$

11.How many friends must you have to guarantee that atleast of them will have birthday in the same month?

12.How many positive integers less than or equal to 1000are divisible either by 3 or 5 or 11.

13.How many integers from 1 and 1000 are divisible by

a. 2 or 3 or 5

b. 2 and 3 but not by 5.

14.A survey of 500 television watchers produced the following information:

285 watch Cricket , 195 watch Hockey , 115 watch Tennis , 45 watch Cricket and Tennis, 70watch Cricket and Hockey , 50 watch Hockey and Tennis , 50 do not watch any of the 3 games.

a. How many people in the survey watch all the 3 games?
b. How many people watch exactly 1 of the 3 games?

15. How many integers are there from 100 to 999 inclusive that are divisible by 3 but not by 4?

16.Find the disjunctive normal form for the function : $f(x,y,z) = x \wedge (y \vee z)$

17.Simplify and write disjunctive normal form of Boolean Expression,

$f(x,y,z) = x \wedge (\overline{y \vee z}) \vee \{[(x \wedge y) \wedge \bar{z}] \wedge x\}$.

19. Find disjunctive normal form of the function

$f(x,y,z) = (x \vee y) \vee [(\overline{\bar{x} \vee y \vee z})]$.

20.Write Boolean Expression $f(x) = (x \vee y) \wedge \bar{z}$ is disjunctive normal form.

21. Let $E(x_1, x_2) = x_1 \wedge (x_1 \vee x_2)$be a Boolean expression over $[L; \wedge \vee, -]$ where L{0,1}.Write $E(x_1, x_2)$ in both disjunctive and conjunctive normal forms.

22. Let $(L, \vee, \wedge)$ be a lattice

a. Commutative of join and meet : For $a, b \in L.\ a \wedge b = a = b \vee a, a \wedge b = b \wedge a$

b. Associativity of join and meet : For $a, b, c \in L.\ (a \vee b) \vee c = a \vee (b \vee c); (a \wedge b) \wedge c = a \wedge (b \wedge c)$

c. Absorption property of join and meet : For $a, b \in L.\ a \vee (a \wedge b) = a\ ;\ a \wedge (a \vee b) = a$.

23. Express the following statements in symbolic form using quantifiers.Also write their negation.

a. No rabbit knows calculus.

b. All Bollywood Movies are serious.

24.Write the truth set of the following precates, if the univers of discourse is U={1,2,3,4,5}

a. $(\exists x), x^2 + 3x + 2 = 0$

b. $(\forall x), (x^2 \geq 9)$

c. $(\forall x), (x\ is\ prime\ integer)$

d. $(\exists x), (x\ is\ divisor\ of\ 13)$

25. Convert the following sentence in to symbolic form and write their nagations.

a. All good politicians are actors.

b. All lazy student do not get good marks.

26. Discuss the validity of the following argument "Either attends the lecture or he watches the movie. If Hari attends the lecture, then he will have a cup of coffee. Hari will go to Hotel, if he watches the movie .Therefore , either Hari will have a cup of coffee or he will go to hotel.(A,B,C,D)".

27. Prove validity of the following argument by using method of indirect proof. "If Meena marries Rohit , she will be in Nashik .If Meena marries Tanmay , she will be in Baramati. If she either in Nashik or Baramati , she will definitely be settled in life. She is not settled in life. Thus she did not Marry Rohit or Tanmay".

28. Define valid argument. Write the following argument in symbolic form." Either I shall read a book or watch a movie. I shall not read a book. Hence I shall watch a movie".

29. Let R be an equivalence relation on a set A. Prove that any two equivalence classes are either disjoint or identical.Also prove that if $a, b \in A, then\ b \in [a] = [b]$.

30. Obtain transitive closure of R defined on set A= {a,b,c,d} by Warshall's algorithm where R={(a,a) , (a,d) , (b,b) , (c,d) , (d,b) , ,(d,d)}

31.Let~ be an equivalence relation on a nonempty set A. Then prove that

a.$a \in [a] \forall a \in A.$

b.$a \in [b] if and\ only\ if [a] = [b] for\ any\ a, b \in A.$

c. Any two equivalence classes are either disjoint or identical.