Anekant Education Society's

Tuljaram Chaturchand College of

Arts, Commerce and Science, Baramati

(Autonomous)

QUESTION BANK

FOR

M.Sc. - SEM-I

STATISTICS

PAPER: STAT-4104

Sampling Theory

Define the following.

1. Population.
2. Sample.
3. Sampling Unit.
4. Sampling.
5. Sampling frame.
6. Sampling design.
7. Non-random Sampling.

SRSWOR

Long answers and theorem.

1. Prove that in simple random sampling, the probability of selecting specified unit of the population at any given draw is equal to the probability of its being selected at the first draw.
2. In simple random sampling the probability that specified unit is included in the sample is $\frac{n}{N}$ where N is the population size and n is the sample size.
3. Show that in simple random sampling without replacement scheme , probability that particular sample selected is $\frac{1}{{}_{N}C_{n}}$.
4. Explain simple random sampling without replacement scheme and show that sample mean is an unbiased estimator of the population mean.
5. Prove that in simple random sampling without replacement scheme, sample mean sum of square is an unbiased estimator of the population mean sum of square.
6. Obtain the expression for the variance of estimator of the population mean in SRSWOR.
7. Prove that in SRSWOR method , the variance of the estimator of the population mean is given by $(\frac{N-n}{Nn})\ S_y^2$ where S_y^2 is the population mean sum of squares.
8. Obtain the confidence interval for the population mean in case of SRSWOR scheme when S^2 is unknown.
9. Obtain the estimator of the population total in case of SRSWOR method . Also obtain the variance of the estimator of the population total.

SRSWR

Long answers and theorem.

1. Explain the simple random sampling with replacement scheme and show that sample mean is an unbiased estimator of the population mean.

2. Give four differentiating points between SRSWOR and SRSWR.
3. Show that in SRSWR , sample mean sum of squares is an unbiased estimator of the population variance .Hence find unbiased estimator of the population mean sum of square.
4. Prove that in case of SRSWR , variance of the estimator of the population mean is given by
5. Prove that in case of SRSWR ,estimator of the variance of estimator of population mean is
6. Obtain the standard error for the estimator of the population mean in SRSWR.
7. Prove that simple random sampling without replacement scheme is always efficient that with replacement scheme.
8. Obtain the estimator of the population total in SRSWR. Also obtain its variance.
9. Construct confidence interval for the population mean in SRSWR scheme.
10. Prove that in SRSWR , probability that specified sample is selected is where N is the population size and n is the sample size.
11. Samplemeanbasedondistinctobservationsisabetterestimatorofthepopulationmeanthanthesamplemeanbasedonallobservations,forSRSWRdesign

Simple Random Sampling for Proportion

Long answers and theorem.

1. Show that in the simple random sampling for the proportion , sample proportion based on sample of size n is an unbiased estimator of population proportion.
2. Show that in the simple random sampling for proportion without replacement scheme , the variance of the estimator of the population proportion is given by
   where P is the population proportion and Q=1-P.
3. Prove that estimator of variance of the sample proportion in SRSWOR is
4. Construct 100(1-α) % confidence interval for the population proportion in case of SRSWOR scheme.
5. What is the difference between inverse sampling and simple random sampling for the proportion. Prove that in case of inverse sample p^= is an unbiased estimator of the population mean , where n is the number of units required to gete specified number of m units with the rare attribute

Determination of sample size.

Long answers and theorem

1. In a simple random sampling without replacement , obtain the minimum sample size if the value of the variance of the estimator of the population mean is prespecified.
2. Estimate the sample size for the specified error in the estimation and the confidence coefficient in case of SRSWOR.
3. Determine the minimum sample size for the SRSWOR if the value of the width of the confidence interval for the population mean is prespecified.
4. Obtain minimum sample size if the value of relative error in the estimation of population mean with confidence coefficient α is specified for the SRSWOR method.

Stratified random Sampling

Long answers and theorem

1. Explain stratified random sampling .Show that sample version of the population eman is a biased estimator of the population mean.
2. Describe the following.
    a) Proportional Allocation.
    b) Equal allocation.
    c) Proportional allocation for given cost.
    d) Optimum allocation.
    e) Neyman's allocation.
3. Describe the proportional allocation for the given cost. Obtain the number of units to be sampled from each stratum in this case. Hence obtain variance of the estimator of the population mean in this allocation.
4. Obtain number of units to be sampled from each startum by using Neyman's allocation. Obtain the variance of the estimator of the population mean in case of Neyman's optimum allocation.
5. Describe optimum allocation for the stratified random sample . Also obtain the variance of the estimator of the population mean in case of optimum allocation.
6. Determine the expression for gain in precision due to stratification.
7. Explain : What is difference between stratification and post stratification .Show that sample mean of post stratified sampling is an unbiased estimator of the population mean.

8. What is post stratification? Give one real life situation where this method is used. Obtain the variance of sample mean of post stratified random sampling scheme.
9. Obtain an unbiased estimator of the population proportion in case of stratified random sampling for proportion.
10. Describe how to construct strata.
11. Describe the collapsed stratification and deep stratification.

Systematic Sampling.

Long answers and theorem

1. Define the linear systematic sampling .Prove that sample mean of the systematic ample is an unbiased estimator of the population mean .Also prove that variance of this estimator is where S is the between systematic sample mean sum of square given by
2. Describe the procedure of linear systematic sampling, Obtain all possible linear systematic samples of size n=6 , if the population size is N=24.
3. Prove that in systematic sampling, variance of estimator of the population mean is given by
4. Prove that systematic samling is more efficient than SRSWOR scheme if within sample mean sum of square is larger than population mean sum of squares.
5. Prove that variance of estimator of the population mean in systematic sampling is where is the intraclass correlation coefficient between the units of the same systematic sample is
6. Prove that systematic sampling is more efficient is more efficient than SRSWOR if where is the intraclass correlation coefficient between the units of the same sample.
7. Compare the systematic sampling and stratified random sampling.
8. Prove that systematic sampling is more precise than SRSWOR in presence of the population possessing linear trend.
9. If the population possessing linear trend and then prove that where denotes the variance of estimator of population mean for stratified , systematic and simple random sampling respectively.
10. Describe the centered systematic sampling. What are the disadvantages of the centered systematic sampling.
11. Describe merits and demerits of systematic sampling.
12. Describe the procedure to obtain the circular systematic sample with an illustration .Give the condition under which all the units in the circular systematic sample are distinct.

13. Explain the difficulties that arise when N≠nk in systematic sampling.
14. Give the reasons why there is need of circular systematic sampling.

Ratio Method and regression method

Long answers and theorem

1. Show that ratio estimator is an unbiased estimator of the population. Hence find biased in the estimation of by using
2. Prove that approximate mean square error of the ratio estimator of the population mean is given by where
3. Show that ratio method of estimator is better than SRSWOR method if where
4. Obtain an unbiased estimator of mean square error of ratio estimator
5. Describe unbiased type ratio estimator. Hence show that where is unbiased type ratio estimator
6. Define JackKnife (Quenouille) ratio estimator of the population mean and show that this estimator is an unbiased estimator of the population mean.
7. Describe the ratio method of estimation for the stratified random sampling.
8. Obtain an expression for regression estimator of the population mean .Show that regression estimator of the population mean is an unbiased estimator. Hence find bias in it.
9. Prove that mean square error of the ratio estimator of the population mean is where is correlation coefficient between auxiliary variable and variable of the interest.
10. Prove that ratio method of estimation is less precise than regression method of estimation.
11. Prove that regression method of estimation is always superior than ratio method and SRSWOR method.

Unequal Probability Sampling.

Long answers and theorem

1. Define the first and second order inclusion probabilities.Hence find the same for the SRSWOR scheme with N population size and n as the sample size
2. Prove that for any sampling design P(.)
3. For any sampling design P(.) , prove that where is the indicator random variable given by

4. Prove that Horvitz-Thompson estimator is an unbiased estimator of the population mean Hence obtain its variance.
5. Explain the cumulative total method of selecting PPS sample with replacement scheme.Also explain the limitation of this method.
6. Describe the Lahiri's method of selecting PPS sample of size n units with replacement scheme.
7. Show that in varying probability sampling scheme with replacement where is an unbiased estimator of population mean and its variance is

Cluster Sampling.

Long answers and theorem

1. What is the difference between cluster sampling and stratified random sampling .Prove that sample mean of cluster means selected in the sample is an unbiased estimator of the population mean in case of cluster having equal size.
2. Obtain the relative efficiency of cluster sampling with respect to simple random sampling .State the conclusion from it.
3. In case of clusters having equal size , prove that where
4. Obtain the expression for relative efficiency of cluster sampling with respect to SRSWOR in terms of intra class correlation coefficient.
5. Explain the effect of cluster size on the relative efficiency.
6. Determine the optimum value of no. of cluster to be selected in the sample in case of cluster sampling with equal cluster sizes.
7. Prove that in case of cluster sampling for the proportion where Pi =Proportion of units in the ith cluster that belongs to given quality characteristic is an unbiased estimator of population mean. Also prove that where
8. Prove that in case of cluster sampling with unequal cluster sizes ,sample mean of cluster means selected in the sample is biased estimator of the population mean .Hence find the bias.
9. Prove the following theorem: For the cluster sampling with unequal cluster size , the estimator where is an unbiased estimator of the population mean.

Double Sampling.

Long answers and theorem

1. Explain the concept of double sampling and obtain the mean squared error of the regression estimator of population mean using double sampling.
2. How double sampling can be used in simple ratio and regression estimator of population mean.
3. Explain the concept of double sampling and discuss how it is used
4. Derive , to the first approximation , variance of the ratio estimator of population mean in double sampling.
5. Show that the ratio estimator $\bar{y}$ rd based on double sampling is biased. Obtain its sampling variance and derive the condition under which estimator $\bar{y}$ rd is more efficient than the estimator based on SRS when no auxiliary information is used.
6. Obtain the optimum variance of ratio estimator in double sampling and optimum variance in srswor for the given cost and find their relative efficiency.

Two stage sampling

Long answers and theorem

1. Explain the method of two stage sampling. Define an unbiased estimator of population mean with equal number of first stage and second stage units. Derive the expression for its variance. Show that two stage sampling is generalization of stratified sampling as well as cluster sampling.
2. State the unbiased estimator of population mean with equal number of first stage and second stage units. Also derive the expression for its sampling variance.
3. Prove that the two stage sampling is better than one stage sampling if $\rho < 0$, where ρ is the intra class correlation between the elements of the first stage units .(equal first stage units)
4. Prove, in usual notations, that the relative efficiency of two stage sampling is compared to one stage sampling approximation to $\frac{1}{1+\rho(m-1)}$ when f. p. c 's at both stages are ignored.

Non sampling errors

Long answers.

1. Distinguish between sampling and non-sampling errors. Discuss in detail the mathematical model for measurement of observational errors
2. Expl ain in brief 'Warner's Randomized Response Technique' (RRT)