# Anekant Education Society's
# Tuljaram Chaturchand College of
# Arts, Commerce and Science, Baramati
# (Autonomous)
**Question bank of M.Sc.**
**STAT-4103: Probability Distributions**

**Choose the correct alternative of the following:-**

1) Let $X$ be a random variable with $pdf\ f(x) = \frac{\theta}{x^{\theta+1}}\ ; x \geq 1, \theta > 0$ then $E(X)$ is equal to …

   a) $\frac{\theta}{\theta-1}\ ; \theta > 1$  b) $\frac{\theta}{\theta+1}\ ; \theta > 1$  c) $\frac{\theta}{\theta-2}\ ; \theta > 2$  d) $\frac{\theta}{\theta+2}\ ; \theta > 2$

2) Let $X$ be a continuous random variable with distribution function $F_X(x)$. Define $Y = F_X(X)$. Then the distribution of $-\log(1 - Y)$ is ….

   a) Standard Normal  b) $\mathrm{U}(0,1)$
   c) Standard Laplace  d) Standard Exponential

3) Let $X$ be a degenerate random variable such that $P(X = 2) = 1$. Then $\mathrm{E(X)} = \cdots$ and $\mathrm{Var(X)} = \cdots$

   a) $1, 2$  b) $1$, does not exists  c) $2, 0$  d) $2, 1$

4) Let $X|p \sim Binomial(n, p)$ and $P \sim Beta(\alpha, \beta)$ then $E(X)$ is
   a) $n\alpha$
   b) $n\beta$
   c) $\frac{n\alpha}{\alpha+\beta}$
   d) $\frac{n\beta}{\alpha+\beta}$

5) Which of the following function is not density function?
   a) $f(x) = \begin{cases} Sin\ X & 0 < x < \frac{\pi}{2} \\ 0 & otherwise \end{cases}$
   b) $f(x) = \begin{cases} \frac{1}{\theta} e^{-\left(\frac{x-\mu}{\theta}\right)} & x > \mu, \theta > 0 \\ 0 & otherwise \end{cases}$
   c) $f(x) = \begin{cases} x(2 - x) & 0 < x < 2 \\ 0 & otherwise \end{cases}$

d) $f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$

6) Let $X \sim \text{Poisson}(m)$ then the distribution of $Y = X^2 + 3$ is

a) $P(Y = y) = \begin{cases} \frac{e^{-m} m^{\sqrt{y-3}}}{\sqrt{(y-3)!}} & y = 3,4,7,12 \ldots \\ 0 & otherwise \end{cases}$

b) $P(Y = y) = \begin{cases} \frac{e^{-m} m^{\sqrt{y-3}}}{\sqrt{(y-3)!}} & y = 0,1,2 \ldots \\ 0 & otherwise \end{cases}$

c) $P(Y = y) = \begin{cases} \frac{e^{-m} m^{y-3}}{(y-3)!} & y = 3,4,7,12 \ldots \\ 0 & otherwise \end{cases}$

d) None of the above

7) Let $X_1$ and $X_2$ are iid random variables with exp(1) then moment generating function of $X_1 - X_2$ is

a) $\frac{1}{1-t^2}$ b) $\frac{1}{1+t^2}$

c) $\frac{1}{1-t}$ d) $\frac{1}{1+t}$

8) Let X be a random variable such that variance of X is $\frac{1}{2}$. Then an upper bound for $P[\,|X - E(X)| > 1]$ as given by the Chebyshev's inequality is

a) $\frac{1}{4}$ b) 1 c) $\frac{1}{2}$ d) $\frac{3}{4}$

9) Let X be a random variable with B (n, p). Then the distribution of n – X is:

a) B (n-1,p) b) B (n,1-p) c) B (n-1,1-p) d) B (n, p)

10) Suppose X has B (n, p) distribution then moment generating function of X is

a) $(p + qt)^n$

b) $(p + qe^t)^n$

c) $(q + pe^t)^n$

d) $(q + pet)^n$

11) Two random variables X and Y are independent if and only if

a) Corr (X, Y) = 0

b) E (X, Y) = E (X). E(Y)

c) E ($e^{txy}$) = E($e^{tx}$). E($e^{ty}$), for all t ∈ R

d) E ($I_{[x \le t]}$. $I_{[y\le s]}$) = E ($I_{[x \le t]}$. $I_{[y\le s]}$) For all t and s in R, where $I_A$ denotes the indicator function of the set A.

12) Let $F$ be a function of two variables defined by

$$F(x,y) = \begin{cases} 0 & if\ x+y<1 \\ 1 & otherwise \end{cases}$$

Which of the following statement is not correct?

a) $F(x,y)$ is non decreasing function.

b) $F(x,y)$ is continuous from right with respect to each coordinate

c) $P\left(\frac{1}{4} < X \le 1, \frac{1}{4} < Y \le 1\right) = 0$

d) $F(\infty,\infty) = 1$

13) Let $(X,Y)$ have the joint pdf $f(x,y) = \frac{1}{4}$ inside the square with corners at the points $(1,1), (-1,1), (1,-1)$ and $(-1,-1)$ in the $(x,y)$ plane and =0 otherwise. Then $P(X^2 + Y^2 < 1)$ is

a)$\frac{\pi}{6}$ b) $\frac{\pi}{8}$ c) $\frac{1}{2}$ d) $\frac{\pi}{4}$

14) If (X,Y)~ Dirichlet (m,n) then marginal distribution of X and Y are

a) Normal distribution b) Exponential distribution

c) Beta distribution of first kind d) Beta distribution of second kind

15) Under the null hypothesis the distribution of sign statistics is

a) Binomial b) multinomial c) Normal d) Chi-square

16) If (X,Y) ~ Bivariate Normal (0,0,1,1,ρ) then distribution of Z=Y given X is

a) Cauchy distribution b) Normal distribution c)F distribution d) t distribution

17) If (X,Y) ~ Bivariate Normal (0,0,1,1,ρ) then correlation coefficient between $X^2$ and $Y^2$ is

a)ρ b)ρ2 c)0 d)+1

18) Let $x_1,x_2$,-----,$x_n$ be independent exponential random variables with respective failure rates $\lambda_1,\lambda_2$,----- $\lambda_n$ then P[x2≤Min($x_1,x_2$,-----,$x_n$)] is

a)∑λi b)λ1 c)$\lambda 2/\sum \lambda i$ d)$\sum \lambda i/\lambda 1$

19) Let X and Y be two independent Poisson random variables with man 1 then P[X+Y=0] is

a)$2e^{-2}$ b)$2e^{-1}$ c)$e^{-2}$ d)$e^{-1}$

20) Let x1,x2,x3,----xn be a random sample from exponential distribution with mean θ then E(X(1)) is

a)θ b) nθ c) $\frac{1}{\theta}$ d)$\frac{1}{n\theta}$

**Unit1**

**Define the terms:**

1) Random Variable
2) Random experiment
3) sample space
4) discrete random variable
5) continuous random variable
6) Moment generating function
7) Probability generating function
8) Distribution function
9) Probability generating function
10) Distribution function

**Questions for 4 marks:**

1) Check whether following function is distribution function. If so find the corresponding probability density function.

$$F(x)=\begin{cases} 0 & ;x<1 \\ \frac{(x-1)^2}{8} & ;1\le x<3 \\ 1 & :x\ge 3 \end{cases}$$

2) Examine following function is cumulative distribution function of a random variable?

$$F(x) = \begin{cases} 1- e^{-x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$

3) Let X be a random variable with probability density function.

$$f(x)=\begin{cases} \frac{1}{2\theta} & ; -\theta \le x \le \theta \\ 0 & ;otherwise \end{cases}$$

Let $Y = \frac{1}{X^2}$ . Find the probability density function of Y.

4) Suppose X has a Cauchy distribution with location 0 and scale 1.Find the distribution of Y=$X^2$

5) Let $X \sim U(0, 2\pi)$. Obtain the $pdf$ of $SinX$.

6) Let $X \sim Exponential(\theta)$. Obtain the distribution of $Y = \left(X - \frac{1}{\theta}\right)^2$

**Question for 6 marks:-**

1) Let X be a random variable with distribution function

$$F(x) = \begin{cases} 0 & ;\ x < 1 \\ 0.3 & ;\ 1 \le x < 2 \\ 0.9 - \frac{2}{x^2} & ;\ 2 \le x < 3 \\ 1 - \frac{2}{x^2} & ;\ x \ge 3 \end{cases}$$

Decompose F(x) as a mixture of discrete and continuous distribution. Also obtain its mean.

2) Let X be a random variable with distribution function

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{5-|X|}{8} & -2 \le x < -1 \\ \frac{8-|X|}{8} & -1 \le x < 0 \end{cases}$$

Decompose F(x) as a mixture of discrete and continuous distribution function

3) Let X be a random variable with distribution function

$$f(x) = \begin{cases} 0 & x < 1 \\ 0.3 & 1 \le x < 2 \\ 0.9 - \frac{2}{x^2} & 2 \le x < 3 \\ 1 - \frac{2}{x^3} & x \ge 3 \end{cases}$$

Decompose F(x) as a mixture of discrete and continuous distribution functions.

4) Consider the following distribution function

$$F(x) = \begin{cases} 0 & if\ x < -2 \\ \frac{1}{3} & if -2 \leq x < 0 \\ \frac{1}{2} & if\ 0 \leq x < 5 \\ \frac{1}{2} + \frac{(x-5)^2}{2} & if\ 5 \leq x < 5 \\ 1 & if\ x \geq 6 \end{cases}$$

Decompose $F$ as a mixture of discrete and continuous distribution function. Find mean of $X$.

5) Let a distribution function $(d.f.)$ be given by $F(x) = \begin{cases} 0 & ; x < 0 \\ \frac{x+1}{2} & ; 0 \leq x < 1 \\ 1 & ; x \geq 1 \end{cases}$

Sketch the given df. Is the given d.f.is mixture d.f.? If so, decompose it in a continuous and discrete d.f . Also find the E(x).

**Question for 8 marks:-**

1) Let $X$ be random variable of the continuous type with PDF $f(x)$. Let $y = g(x)$ be differential for all $x$ and either $g'(x) > 0$ for all $x$ or $g'(x) < 0$ for all $x$. Then prove that the probability density function for the random variable $Y = g(X)$ is

$$h(y) = \begin{cases} f\left(g^{-1}(y)\right)\left|\frac{d}{dy} g^{-1}(y)\right| & \alpha < y < \beta \\ 0 & otherwise \end{cases}$$

where $\alpha = \min\{g(-\infty), g(+\infty)\}$ and $\beta = \max\{g(-\infty), g(+\infty)\}$

2) Find the probability generating function of the random variable X ~ B (n, p) and hence obtain mean and variance. If Y ~ B (m, p) and X, Y are independent random variable, find the probability distribution of X +Y.
3) Define probability generating function (p.g.f.) of r.v.X. Obtain p.g.f. of Poisson random variable.
4) If $x_1, x_2, x_3, \text{----}, x_N$ are iid random variables with common PGF $P_x(s)$ and N is random variable with PGF $Q_N(.)$ then show that PGF of $S_N = x_1+x_2+x_3+\text{----}+x_N$ is $Q_N(Px(s))$
5) If $x_1, x_2, x_3, \text{----}, x_N$ are iid Bernoulli (p) and N is random variable with Poisson(λ) then find the probability distribution of $S_N = x_1+x_2+x_3+\text{----}+x_N$

# Unit 2

Define the following terms:-

1) Multiple random variable
2) Bivariate random vector (x,y)
3) joint probability distribution
4) marginal probability distribution
5) Conditional probability distribution
6) Conditional expectation
7) Convolution of random variable
8) Compound distribution
9) Location-scale family
10) Location family
11) Non regular family
12) Multiple correlation function
13) Partial correlation coefficient

**Question for 4 marks:-**

1) If E(Y) exists, then show that E(Y) = E [ E (Y|X)] in discrete and continuous case.

2) Prove that:

   i) Let $E\big(h(X)\big)$ exists. Then $E\big(h(X)\big) = E\{E(h(X)|Y)\}$.

   ii) If $E(X^2) < \infty$ then $var(X) = var(E\{X|Y\}) + E(var\{X|Y\})$.

3) Let $S_N = \sum_{i=1}^{N} Xi$ where N is a Poisson variable with parameter 1 and $X_i$'s are independent and identically distributed Bernoulli variables with parameter p. Find expected value and variance of $S_N$.

4) Let X be a random variable with probability density function,

$$f(x) = \begin{cases} 2(1-x) & ; 0 < x < 1 \\ 0 & ; otherwise \end{cases}$$

   Sketch the graph of f(x).

5) State the necessary and sufficient condition for a function F (X, Y) to be a bivariate c.d.f.

6) Let X be random variable with pdf

$$F(x) = \begin{cases} \tfrac{1}{2} e^{-x/2} & x \geq 0 \\ 0 & \text{other wise} \end{cases}$$

Find m.g.f. and hence find mean and variance of X.

7) Let $X \sim U(0,1)$, $Y \sim U(0,1)$ and $X$ and $Y$ are independent. Use method of convolution to find the density of $X+Y$.

8) If X & Y are iid Exp(λ) using convolution find the probability distribution of X+Y.

9) If X & Y are iid Exp(1) using convolution find the probability distribution of X+Y.

10) Define compound distribution and obtain its mean and variance.

11) Suppose $C$ denotes the unit circle in the plane. $C = \{(x,y): x^2 + y^2 \leq 1\}$. We pick a point $(X,Y)$ at random from $C$.

i) Find $c$ such that $f(x,y) = \begin{cases} c & ;(x,y) \in C \\ 0 & otherwise \end{cases}$.

ii) Find the marginal densities for $X$ and $Y$ and the conditional densities.

12) Obtain the Characteristic function for $U(-1,1)$ distribution.

13) Let $X$ be random variable with $pdf$

$$f(x) = \begin{cases} \frac{x}{2} & if\ 0 \leq x < 1 \\ \frac{1}{2} & if\ 1 < x < 2 \\ \frac{1}{2}(3-x) & if\ 2 < x \leq 3 \end{cases}$$

Show that moment of all order exists. Find mean of $X$.

14) Let $X$ have the triangular $pdf$ $f(x) = \begin{cases} x & 0 < x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$. Show that given $pdf$ is symmetric at 1. Also find $E(X)$.

15) In a trivariate population with variable $X_1, X_2$ and $X_3$ it is given that the simple correlation between any two variables is equal to $\frac{1}{2}$. Find $R_{1.23}$ and $r_{12.3}$.

**Questions for 8 marks:-**

1) The $joint\ pmf$ of $(X,Y)$ is given by

$$P(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!\,(x-y)!}\ ;\ y = 0,1,\ldots x, x = 0,1,\ldots, 0 < p < 1, \lambda > 0$$

Find marginal distribution of $Y$ and find conditional distribution of $X|\ Y = y$.

2) State trinomial distribution for $(X,Y)$ and state the results related to it.

3) Let X and Y be continuous random variable with joint probability density function,

$$f(x,y)=\begin{cases}21x^2y^3 & ;\ 0<x<y<1\\ 0 & ;\ otherwise\end{cases}$$

i) Find the marginal probability distribution of X and Y.

ii) Find conditional probability distribution of X|Y = y and E [X|Y=y].

4) If (X, Y) is a random vector with probability density function,

$$f(x,y)=\begin{cases}e^{-(x+y)} & ;\ x>0, y>0\\ 0 & ;otherwise\end{cases}$$

Find m.g.f. of (X, Y). Hence find marginal m.g.f. of X and Y. Also verify whether X and Y are independent. Obtain the m.g.f. of X+Y using joint m.g.f.

5) Let (X, Y, Z) be random vector with probability density function,

$$f(x,y,z)=\begin{cases}\dfrac{6}{(1+x+y+z)^4} & ;\ x>0, y>0, z>0\end{cases}$$

Obtain probability density function of X+Y+Z

6) Let f (x,y) = $\begin{cases}8xy & ;\ 0<x<y<1\\ 0 & ;\ otherwise\end{cases}$

Find i) E(Y/ X=x) ii) Var (Y/ X=x)

7) Let f(x, y) = 4x ( 1 – Y) 0< x < 1 ; 0 < y < 1

0 other wise

Obtain

i) Marginal distribution of X and Y.

ii) Conditional distribution of X given Y = y.

8) Let $X_1, X_2, \ldots, X_k$ are $k$ independent Poisson variates with parameters $\lambda_1, \lambda_2, \ldots, \lambda_k$ respectively. Show that the conditional distribution of $X_1, X_2, \ldots, X_k$ given $\sum_{i=1}^{k} X_i = x$ is multinomial.

9) Let $f(x,y,z)=\begin{cases}e^{-x-y-z} & ;x>0, y>0, z>0\\ 0 & otherwise\end{cases}$ be the joint $pdf$ of $(X,Y,Z)$. Compute $P(X<Y<Z)$ and $P(X=Y<Z)$.

10) The $joint\ pmf$ of $(X, Y)$ is given by

$$P(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!\,(x-y)!}\ ;\ y = 0,1,\dots x, x = 0,1,\dots, 0 < p < 1, \lambda > 0$$

Find marginal distribution of $Y$ and find conditional distribution of $X|\ Y = y$.

## Unit 3

**Define the following terms:-**

1) Bivariate Normal random variable (x,y)
2) Bivariate Poisson  random variable(x,y)
3) Bivariate exponential family
4) Dirichlet distribution

**Questions for 4 marks:-**

1) Show that P (min($T_1$,$T_2$)≤t)=1-$e^{-\theta t}$ where($T_1$,$T_2$) follow bivariate exponential distribution.
2) Define Dirichlet distribution .Show that bivariate beta distribution is a special case of Dirichlet distribution.
3) Define bivariate exponential distribution.state the memory less property satisfied by this distribution.
4) Define Dirichlet distribution .Obtain conditional distribution of Y given X where(x,y) follows Dirichlet distribution. Identify this distribution.
5) Define bivariate Poisson random variable (x,y). Obtain conditional distribution of Xgiven Y=y
6) Define the Dirichlet distribution. Obtain marginal distribution of Y where (x,y)follows Dirichlet distribution.
7) Define Bivariate Normal distribution.obtain its m.g.f.
8) Define Bivariate exponential distribution (Marshall Olkin’s method).Prove that it satisfies forgetfulness property.

**Questions for 6 marks:-**

1) Define Bivariate Normal distribution.Obtain its m.g.f.

2) State the regularity condition of exponential family.Check whether the following distributions are belongs to exponential family.

a) X~exp(θ)
b) X~N(θ,1)
c) X~P(θ)
d) X~G(α,β),both α and β are unknown.
e) X~C(1,θ) ,where θЄR
f) X~NB(k,p) when k & p are known.
g) X~U(0,θ)

3) Define Bivariate Poisson distribution. Derive its m.g.f.

4) If X & Y follow univariate normal distribution does(X,Y) always follow bivariate normal distribution? Justify your answer.

**Questions for 8 marks:-**

1) Define bivariate normal distribution.If(X,Y) follow bivariate normal distribution.obtain m.g.f.of (X,Y). Show that X & Y are independent iff $\rho=0$

## Unit 4

**Define the following terms:-**

1) Non-central chi-square distribution
2) Non-central F distribution
3) Order statistics
4) Non central t-distribution
5) Quadratic form

**Questions for 4 marks:-**

1) State and prove probability integral transformation theorem.
2) Write note on distribution free test.

3) Let $x_1, x_2$,-----$x_n$ be a random sample from U(0,1) Find distribution of range.

4) Explain the terms:

   a) Distribution free statistics

   b) Empirical distribution function

6) Let A be an n × n symmetric matrix and Q = $\underset{-}{X}' \underset{-}{A} \underset{-}{X}$ obtain m.g.f. of Q

7) Let $x_1, x_2$,-----,$x_n$ be a random sample from U(0,1) Obtain probability distribution of i)$x_{(1)}$ ii)$x_{(2)}$

8) Define Wilcoxon sign rank test for the population median state its test statistics.

9) Describe Kolmogorov –smirnov test.Prove that Dn+ is distribution free.

10) Let $x_1, x_2$,-----, $x_n$ be independent and identically distributed random variables from continuous distribution. Find the joint distribution of $r^{th}$ & $s^{th}$ order statistic.

11) Define W, the Wilcoxon statistic for testing the equality of two continuous distribution functions. Obtain mean and variance of W under the null hypothesis.

12) Let $x_1, x_2$,-----,$x_n$ be a random sample from U(0,1) Obtain probability distribution of $x_{(r)}$ ,$x_{(n-1)}$

13) Define Order statistics corresponding to a random sample of size n from continuous probability distribution .Obtain the joint probability distribution of $n^{th}$ order statistics.

14) Define Wilcoxon sign rank test for the population median. Obtain the null probability distribution of its test statistics.

15) Define order statistics.Let $x_1, x_2$,-----, $x_n$ be independent and identically distributed random variables from continuous distribution. Find the joint distribution of $r^{th}$ & $s^{th}$ order statistic.

16) Define order statistics.Let $x_1, x_2$,-----, $x_n$ be independent and identically distributed random variables from continuous distribution. Find the joint distribution of $n^{th}$ order statistic.

**Questions for 8 marks:-**

1) State and prove Fisher- Cochran theorem. Discuss its one application.
2) Derive the probability desity function of non-central t distribution. Also state its mean and variance.

3) Derive the probability desity function of non-central $\chi$2distribution. Also state its mean and variance.
4) Derive the probability desity function of non-central Fdistribution.
5) Let X be a random vector with N(0,In) distribution .show that two quadratic formsc X'AX & X'BX are independent iff AB=0 where A &B are symmetric idempotent matrices. Is the converse true?