

**Anekant Education Society's**  
**Tuljaram Chaturchand College of Arts, Science and Commerce,**  
**Baramati**  
**Autonomous**

**QUESTION BANK**

**FOR**

***F.Y.B.Sc (Computer Science)***

***SEM-I***

**STATISTICS**

**PAPER-II: CSST-1102**

**PROBABILITY**

**AND**

**SOME DISCRETE PROBABILITY DISTRIBUTIONS**

(With effect from June 2019)

## PAPER-II: CSST-1102

### PROBABILITY AND SOME DISCRETE PROBABILITY DISTRIBUTIONS

#### UNIT-1: SAMPLE SPACE AND EVENTS

##### A) Questions for 1 mark:-

##### I] Choose the correct alternative:-

- Which of the following is not a random experiment?
  - Number of runs scored by Sachin Tendulkar are noted in an over bowled by Shoaib Akhtar.
  - You watch T.V. for five hours on the day of the examination , and whether you pass or fail is noted.
  - You tie a friendship band to your friend who is your friend indeed!
  - When you walk on the ground, the earth pushes you.
- Relative Complement of A w.r.t. B is given by
  - $A \cap B'$
  - $A' \cup B$
  - $A' \cap B$
  - $(A \cap B)'$
- In an experiment of planting four seeds, the number of seeds germinated after a week are recorded. The Sample space of this experiment is
  - $(0, 4)$
  - $\{1, 2, 3, 4\}$
  - $[0 4]$
  - $\{0, 1, 2, 3, 4\}$
- If  $A \cap B = \phi$  then the two events A and B are
  - Mutually exclusive
  - Exhaustive events
  - Independent events
  - Dependent events
- If two events A & B are mutually exclusive , then  $P(A \cup B) =$  -----
  - $P(A)P(B)$
  - $P(A) + P(B)$
  - $P(A \cap B)$
  - $P(A) + P(B) - P(A \cap B)$

6. Every subset of a sample space is known as an -----
- Sample space
  - Super set
  - Event
  - An experiment
7. If A is an event then the conditional probability of A given that event  $A^c$  has occurred is
- 0.5
  - 1
  - 0
  - 0.05
8. When a card is drawn from the standard pack of playing cards, which of the following is a pair of disjoint events ?
- An ace & an odd denomination
  - A heart and a queen
  - A club and a red card
  - An even denomination & spade
9. The probability of an impossible event is
- 1
  - $1-P(\phi)$
  - $P(\Omega)$
  - 0
10. If  $\Omega = \{ H, TH, TTH, TTTH, \dots \}$  then sample space is :
- Finite sample space
  - Uncountable infinite sample space
  - Countably finite sample space
  - Countably infinite sample space

**II] State whether each of the following statement is True or False.**

- A and B are mutually exclusive events if and only if  $A \cap B = \phi$ .
- A and B are exhaustive events if and only if  $A \cup B = \Omega$ .
- The event  $\Omega$  and  $\phi$  are mutually exclusive.
- A ball is thrown in the air it will fall down is the example of deterministic experiment.
- Conditional definition of probability is  $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- An event doesn't containing all the points of sample space is called sure event.

**B) Questions for 2 marks:-**

1. Define random experiment.
2. Classify the following experiments into deterministic and Non – deterministic experiment.
  - i) A coin is tossed to decide the team which would bat first in a cricket match.
  - ii) Marketing manager conducts market survey to measure the effect of advertisement on sales.
  - iii) Water is heated up to  $100^{\circ}\text{C}$ .
3. Define sample space and give one example of it.
4. Write down sample space for the following experiment. Also mention its type.
  - i) T.V viewers were asked to give ratings to 3 programmes.
  - ii) Answers to an objective question which has four multiple choices A, B, C, D. Student ticks a single answer.
  - iii) Number of tossing of a die till 6 appear for the first time.
  - iv) A two digit number is formed from the digits 4, 5, 6 using each digit only once.
5. Define an event and give one example of it.
6. Give two examples of countably infinite sample space.
7. Give one example of sure event and impossible event.
8. Define mutually exclusive events.
9. Define mutually exhaustive events.
10. What do you understand by equally likely events?
11. Define elementary event.
12. A coin is tossed till head occurs for the first time. Write down the sample space of this experiment.

**C) Questions for 4 Marks:-**

1. Define sample space. Explain types of sample spaces giving examples of each.
2. What do you mean by an experiment? What are different types of experiments? Explain.
3. Define with example each of the following
  - i) Sample space
  - ii) Discrete sample space
  - iii) Event
  - iv) Elementary event
4. Explain the concept of mutually exclusive and exhaustive events.
5. Write down the sample spaces for following experiment. Also state the type of the sample spaces.
  - i) A sample is taken to predict the result of the particular assembly poll.

- ii) A three digit number is formed from the 5 distinct numbers, using each digit only once.
6. Distinguish clearly between deterministic and non- deterministic experiments. Give suitable examples.

**D) Questions for 6 Marks:-**

1. Explain with one illustration
  - i) Complement of an event
  - ii) Mutually exclusive events
  - iii) Exhaustive events
2. Suppose three fair coins are tossed simultaneously. Let A be the event that exactly 2 coins show heads and B be the events that at least 2 coins show heads. List the elements of A, B, A' and B'. Verify whether A and B are i) mutually exclusive ii) exhaustive events.
3. Two fair dice are rolled. Let A be the event that sum of the points on the uppermost faces is even and B be the event that the product of the two numbers is not greater than 9. List the elements contained in the events- i)  $A \cup B$  ii)  $A \cap B$  iii)  $(A \cap B') \cup A'$

## UNIT-2. PROBABILITY

### A) Questions for 1 mark:-

#### I] Choose the correct alternative:-

1. In the simultaneous tossing of two fair coins, the probability of having one head is  
a) 0.5                      b) 0.25                      c) 0.75                      d) 1
2. Let A be event defined on sample space,  $\Omega$ . Which of the following statement is true?  
a)  $P(A) = 1$     b)  $P(A) = 0$     c)  $0 \leq P(A) \leq 1$     d)  $0 \leq P(A) < 1$
3. If  $A \cap B = \phi$  then the two events A and B are  
a) Mutually exclusive    b) Exhaustive events  
c) Independent events    d) Dependent events
4. Suppose  $\Omega = \{w_1, w_2, w_3, w_4\}$ ;  $P(w_1) = \frac{1}{7}, P(w_2) = k, P(w_3) = k, P(w_4) = \frac{3}{7}$  for what value of k will this be a probability model?  
a) 0                      b) 1                      c)  $\frac{3}{7}$                       d)  $\frac{2}{7}$
5. If A and B are two events defined on  $\Omega$  such that  $A \subset B$  then  
a)  $P(A) = P(B)$     b)  $P(A) < P(B)$     c)  $P(A) \leq P(B)$                       d)  $P(A) > P(B)$
6. If  $P(A) = P(B)$  then the two events are  
a) Equally likely  
b) Dependent  
c) Independent  
d) Both a and b
7. When a card is drawn from the standard pack of playing cards, which of the following is a pair of disjoint events?  
a) An ace and an odd denomination  
b) A heart and a queen  
c) A club and a red card  
d) An even denomination and spade

8.  ${}^n P_r = \dots\dots$

- a)  $\frac{n!}{r!(n-r)!}$       b)  $\frac{n!}{(n-r)!}$       c)  $\frac{n!}{r!}$       d)  $\frac{n!}{n!(n-r)!}$

9. The probability of an impossible event is

- a) 1      b)  $1-P(\phi)$       c)  $P(\Omega)$       d) 0

10. If  $A \subset B$ , then the relation between  $P(A)$  and  $P(B)$  is :

- a)  $P(A) \neq P(B)$       b)  $P(A) = P(B)$       c)  $P(A) > P(B)$       d)  $P(A) \leq P(B)$

11. If  $P(A) = 0.8$ ,  $P(B) = 0.7$ ,  $P(A \cup B) = 0.96$  then  $P(A \cap B)$  is

- a) 0.56      b) 0.06      c) 0.6      d) 0

12. The probability of drawing one red ball randomly from a bag containing 5 red, 7 black and 10 yellow balls is

- a) 0      b)  $1/5$       c)  $5/22$       d)  $1/22$

13. If A and B are independent events with  $P(A) = 0.4$  and  $P(B) = 0.25$  then  $P(A \cup B)$  is

- a) 0.55      b) 0.65      c) 0.1      d) 0.01

14. If A and B are independent events with  $P(A) = 0.4$ ,  $P(B) = 0.5$  then  $P(A \cap B)$

- a) 0.03      b) 0.9      c) 0.1      d) 0.3

15. In conditional probability distribution of Y given  $X=x$ ,

- a) X is variable      b) X is constant      c) Y is variable      d) Y is constant.

16. Which of the following statement is true?

- a) A and  $A'$  form partition of  $\Omega$   
 b) A and  $\Omega$  form partition of  $\Omega$   
 c) A and  $A'$  do not form partition of  $\Omega$   
 d) Only two events cannot form partition of  $\Omega$

17. If A is an event defined on  $\Omega$  then  $P(A | A')$  is ,

- a) 1      b)  $P(A)$       c) 0      d)  $P(A')$

18. Which of the following statement is true ?

- a)  $P(A/B) \geq P(A)$       b)  $P(A/B) \leq P(A)$   
 c)  $P(A/B) = P(A)$       d) nothing can be said about magnitudes of  $P(A)$  and  $P(A/B)$

19. If A is an event then conditional probability of A given that A has already occurred is
- 1
  - 0
  - 0.5
  - 0.75

**II] State whether each of the following statement is True or False.**

- $P(\Omega)=1$  is one of the axioms of probability.
- Probability of drawing a card of king from 52 playing cards is  $1/4$ .
- In tossing a fair coin twice, Probability of getting two heads is  $1/2$ .
- Independence implies mutual exclusiveness.
- Number of combinations of n elements is  $\frac{n!}{(n-r)!}$ .
- Number of permutations of n distinct elements without repetition is  $\frac{n!}{(n-r)! \cdot r!}$ .
- ${}^n C_n = 1$
- $P(\phi) = 0$  is one of the axioms of probability.
- If  $A \subset B$  then  $P(A | B) = 1$
- If A and B are independent then A' and B' are also independent events.
- Baye's' theorem is used to calculate posterior probabilities of events.

**B) Questions for 2 marks:-**

- Give the classical definition of probability.
- Define sample space.
- State axioms of probability.
- Prove that the probability of an impossible event is zero i.e.  $P(\phi) = 0$ .
- Two cards are drawn from a well shuffled pack of playing cards. Find the probability that both cards are of diamond.
- Four cards are drawn from a well shuffled pack of playing cards. Find the probability that each card is of different suit.
- Suppose a pair of fair dice is thrown. Find the probability that both the faces are same.

8. Suppose a pair of fair coin is tossed. Find the probability that at least one coin shows head.
9. For events A and B if  $P(A) = P(A/B) = 0.15$  and  $P(B/A) = 0.20$ , then check whether A and B are independent.
10. State Bayes' theorem.
11. Define independence of two events A and B on  $\Omega$ .
12. A and B are two events defined on a sample space  $\Omega$  state the nature of relationship between A and B if (a)  $P(A/B) = 0$  and (b)  $P(A/B) = P(A)$ .

**C) Questions for 4 marks:-**

1. What are the limitations of classical probability?
2. Define i) sample space ii) probability of an event
3. Define independence of two events A and B on  $\Omega$ . Give an illustration.
4. Does independence of two events imply that the events are mutually exclusive? Justify.
5. Does mutually exclusiveness of two events imply independence? Justify.
6. A and B are two events defined on  $\Omega$  such that  $A \subseteq B$ , then show that  $P(A) \leq P(B)$
7. State and prove addition theorem of probability for two events.
8. Find the total number of permutations of the letters of the word "INDIAN".
9. Urns I and II respectively contain 5 white, 5 black and 2 white, 8 black balls. An urn is selected at random and a ball is drawn at random from the selected urn. If the ball drawn is white, then find the probability that the selected urn was II.
10. Bag I contains 6 blue and 4 red balls. Bag II contains 2 blue and 6 red balls. Bag III contains 1 blue and 8 red balls.
  - i) A bag is chosen at random; a ball is drawn randomly from this bag. It turns out to be blue. Find the probability that bag I was chosen.
  - ii) A bag is chosen at random; two balls is drawn randomly from this bag without replacement from this bag. Both the balls were blue. Find the probability that bag II was chosen.
11. An explosion in a factory manufacturing explosives can occur due to (i) short circuit (ii) defects in machinery (iii) negligence of workers (iv) sabotage. The probabilities of these four causes are known to be 0.3, 0.2, 0.4 and 0.1 respectively. The engineers feel that an explosion can occur with probability (i) 0.3 if there is a short circuit, (ii) 0.2 if there are

- defects in machinery,(iii)0.25 if the workers negligent and (iv)0.8 if there is a sabotage .Given that an explosion has occurred , determine the most likely cause of it.
12. Consider the experiment of rolling two fair dice. Let A= odd number on the first die, B= odd number on the second die, and C= sum of two points is odd. Check whether A, B, and C are pair wise independent and mutually independent.
  13. Consider a pack of playing cards without face cards. 4 cards from this pack are drawn at random. Find the probability that they belong to
    - i) Different suits
    - ii) 2 are face cards
  14. In a certain school, examination results showed that 20% students failed in Mathematics, 5% failed in English while 10% failed in both Mathematics and English. Are the two events failing in Mathematics and failing in English independent. ?
  15. Two urns identical in appearance contain respectively 3 white and 2 black balls, and 2 white and 5 black-balls. One urn is selected at random and a ball is drawn from it. What is the probability that it is black?
  16. A, B, C form a partition of  $\Omega$  .If  $3 P (A) = 2 P (B) = P (C)$  , find  $P(A \cup B)$ .
  17. Prove that:-
    - i)  $P(A') = 1 - P(A)$  where A' is the complement of A.
    - ii) For any event A of  $\Omega$  ,  $0 \leq P(A) \leq 1$ .
  18. In a random arrangement of the letters of the word "DREAM" find the probability that:
    - i) All the vowels come together
    - ii) The arrangement starts with D and ends with M
  19. Given that  $P(A_1) = P(A_2) = P (A_3) = 1/3$  and  $P(B/A_1)=2/7$ ,  $P(B/A_2) = 4/9$ ,  $P (B/A_3) = 1/5$ , find  $P (A_1/B)$ .
  20. In a random arrangement of the letters of the word "STATISTICS", find the probability of, all vowels occupy even places.
  21. How many 3 letters words can be arranged using the letters of the word "SEMINAR".
  22. How many 3 digits numbers can be formed from the six digits 1,3,5,6,7,9 .  
If each digit is to be used only once. Among these how many will be divisible by 5.
  23. How many 3-digit numbers divisible by 5 can be formed out of 3,4,5,6,7 if
    - i) no digit is to be repeated?
    - ii) a digit may be repeated any number of times?

**D) Questions for 6 marks:-**

- 1) State and prove addition theorem of probability for three events.
- 2) If  $A_1, A_2, \dots, A_n$  are events defined on  $\Omega$  then show that  $P(\cup A_i) \leq \sum P(A_i)$
- 3) Let  $A, B, C$  be any three events defined on  $\Omega$ , such that
$$P(A) = 3/8, P(B) = P(C) = 1/4, P(B \cap C) = 0,$$
$$P(A \cap B) = 1/8 = P(A \cap C)$$
Evaluate i)  $P(A \cup B \cup C)$  ii)  $P(A \cup C)$  iii)  $P(A' \cap B' \cap C')$
- 4) Define partition of a sample space. State and prove Baye's theorem.
- 5) Let  $A$  and  $B$  be two events on  $\Omega$ , such that  $P(A) = 3/4$  and  $P(B) = 5/8$   
Prove that i)  $P(A \cup B) \geq 3/4$   
ii)  $3/8 \leq P(A \cap B) \leq 5/8$   
iii)  $1/8 \leq P(A \cap B) \leq 3/8$
- 6) If  $A$  and  $B$  are two independent events defined on  $\Omega$ ; then prove that.
  - (a)  $A$  and  $B'$  are independent,
  - (b)  $A'$  and  $B$  are independent
  - (c)  $A'$  and  $B'$  are independent
- 7) Of the three events  $A, B$  and  $C$ .  $A$  and  $B$  are mutually exclusive,  $A$  and  $C$  are independent.  $B$  and  $C$  are independent. If  $P(A) = 1/4, P(B) = 1/3, P(C) = 1/6$ . Find
  - (i)  $P(A \cup B)$
  - (ii)  $P(A \cap C)$
  - (iii)  $P(A \cup B \cup C)$ .
- 8) A husband and wife appear for two vacancies in the same post. The probability of husband's selection is  $1/7$  and that of wife's selection is  $1/5$ . What is the probability that-
  - (a) Both of them will be selected?
  - (b) Only one of them will be selected?
  - (c) None of them will be selected?
- 9) In a bolt factory, three machines  $A, B, C$  produce respectively 25%, 35% and 40% of the days production. Of the total of their output 5%, 4% and 2% are defective bolts respectively from  $A, B$  and  $C$ . A single bolt is drawn at random from the days' production and is found defective. Determine the probabilities that it was manufactured by machines  $A, B$  and  $C$ .
- 10) Bag I contains 3 blue and 4 red balls. Bag II contains 2 blue and 6 red balls. Bag III contains 1 blue and 8 red balls. A bag is chosen at random; a ball is drawn randomly from the bag. It turns out to be blue. Find the probability that bag – I was chosen.

- 11) Consider that there are three identical bags, A, B and C. The bag A contains 2 gold coins bag B contains 2 silver coins and bag C contains 1 silver and 1 gold coin. What is the probability of selecting bag A out of the three given that a gold coin is selected?
- 12) The letters of the word 'REGULAR' are arranged at random. Find the probability that :
- i) vowels may occupy the even places
  - ii) all vowels are together
  - iii) vowels are never together
- 13) The letters of the word 'COMMERCE' are arranged at random. Find the probability that :
- i) vowels may occupy the even places
  - ii) all vowels are together
  - iii) vowels are never together
- 14) The letters of the word 'COSTING' are arranged at random. Find the probability that the word so formed :
- i) starts with T
  - ii) ends with N
  - iii) starts with T and ends with N

**UNIT 3. Discrete Random Variable**

**A) Questions for 1 mark:-**

**I] Choose the correct alternative**

1. If X and Y denotes the points obtained when two six faced unbiased dies are thrown then  $P(X=Y)$  is
  - a)  $1/2$
  - b)  $1/6$
  - c)  $1/24$
  - d)  $1/36$
2. Which of the following is not a discrete random variable?
  - a) Number of students present in the class.
  - b) Number of persons possessing 'O-ve' blood group in a blood donation camp.
  - c) Number of daughters born to a couple until they get son.
  - d) Weight of a new born baby.
3. Suppose the values of distribution function  $F(x)$  at  $X= x_i$  are as given below:

X	0	1	2	3	4	5	6
F(x <sub>i</sub> )	0.2	0.3	0.5	0.65	0.75	0.9	1

- What is  $P(X=2)$  ?
- a) 0.5
  - b) 0.2
  - c) 0
  - d) cannot be determined
4. The p.m.f. of discrete random variable X is given by :-

X	1	2	3	4	5
P(x)	0.1	0.25	0.25	0.2	0.2

- What is  $P(2 < X < 5)$  ?
- a) 0.9
  - b) 0.5
  - c) 0.45
  - d) 0.3

**B) State whether the following statements are true or false.**

1. A discrete random variable assumes only finite number of values.
2. A discrete random variable is defined on a discrete sample space.
3. A discrete random variable cannot take negative values.
4. Sum of probabilities of all values of a random variable is less than one.
5. The p.m.f. of a discrete random variable is a non decreasing function.
6. We cannot find probability of interval for discrete variable.
7. A distribution function is an increasing function.
8. The distribution function of a discrete random variable is also known as step function.

**B) Questions for 2 marks:-**

1. Explain the following terms:-
  - i) Random Variable and Discrete Random Variable
  - ii) probability mass function (p.m.f.)
  - iii) Probability distribution of a discrete random variable
  - iv) Expectation of function of discrete random variable
  - v) Variance and Standard deviation of random variable
2. Suppose we toss a biased coin twice. Probability of getting head is twice that of getting tail for this coin. What will be the probability of getting no head?
3. Determine k such that the following function is p.m.f.  
 $P(x) = kx$  ;  $x=1,2,3,-----,10$ .

**C) Questions of 4 marks:-**

1. Let the probability distribution of a discrete random variable X be

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

- i) Find k
- ii)  $P[X \geq 2]$
- iii)  $P[0 < X < 5]$
- iv) Obtain distribution function of X.

2. Given the following distribution function of random variable X

X	-3	-2	-1	0	1	2
F(x)	0.04	0.23	0.56	0.82	0.93	1.00

- i) Find  $P[-1 \leq X \leq 2]$
  - ii) Find  $P[X \geq 0]$
  - iii) Find the probability distribution of X
  - iv)  $P[X \geq 1 | X > -1]$
3. Determine k such that the following functions are
- i)  $f(x) = k(2x - 1)$  ;  $x=0,1,2,3,4,5,6$ .
  - ii)  $f(x) = k(x^5)$  ;  $x=0,1,2,3,4,5$ .
4. Number of hardware failures (X) of a computer system in a week of operation has the following p.m.f.

X	0	1	2	3	4	5	6
P(x)	0.18	0.28	0.25	0.18	0.06	0.04	0.01

Find  $E(X)$  and  $V(X)$ .

5. Verify whether the following can be looked upon as p.m.f. for the given values of x.

$$p(x) = \frac{x+1}{10} \quad ; x=0,1,2,3.$$

6. Verify whether the following can be looked upon as p.m.f. for the given values of x.

$$p(x) = \frac{x^2}{30} \quad ; x=0,1,2,3,4.$$

7. Verify whether the following can be looked upon as p.m.f. for the given values of x.

$$p(x) = \frac{x-2}{5} \quad ; x=1,2,3,4,5.$$

8. Verify whether the following can be looked upon as p.m.f. for the given values of x.

$$p(x) = \frac{x^2}{14} \quad ; x=1,2,3.$$

9. Verify whether the following can be looked upon as p.m.f. for the given values of x.

$$p(x) = \frac{x-1}{2} \quad ; x=0,1,2.$$

10. Define cumulative distribution function (c.d.f.) of a discrete random variable and state its important properties.
11. For the following c.d.f.  $F(x)$  of discrete random variable  $X$  obtain the p.m.f. of  $x$ .

$$F(x) = \begin{cases} 0 & ; x < 0 \\ 0.2 & ; 0 \leq x < 2 \\ 0.5 & ; 2 \leq x < 4 \\ 0.7 & ; 4 \leq x < 6 \\ 0.8 & ; 6 \leq x < 8 \\ 1 & ; x \geq 8 \end{cases}$$

12. For the following c.d.f.  $F(x)$  of discrete random variable  $X$  obtain the p.m.f. of  $x$ .

$$F(x) = \begin{cases} 0 & ; x < 1/4 \\ 1/8 & ; 1/4 \leq x < 1/2 \\ 1/4 & ; 1/2 \leq x < 3/4 \\ 1/2 & ; 3/4 \leq x < 5/4 \\ 3/4 & ; 5/4 \leq x < 1 \\ 1 & ; x \geq 8 \end{cases}$$

13. For the following c.d.f.  $F(x)$  of discrete random variable  $X$  obtain the p.m.f. of  $x$ .

$$F(x) = \begin{cases} 0 & ; \text{if } x < 5 \\ \frac{1}{8} & ; 5 \leq x < 10 \\ \frac{7}{24} & ; 10 \leq x < 15 \\ \frac{2}{3} & ; 15 \leq x < 20 \\ \frac{11}{12} & ; 20 \leq x < 25 \\ 1 & ; x \geq 25 \end{cases}$$

14. A random variable  $x$  takes values 0,1,2,3,4 such that  $P(1 < X \leq 4) = 0.55$  ;  $P(X \leq 1) = 0.25$ ;  $P(X=2) = 2P(X=1)$  and  $P(0 < X \leq 2) = 0.45$ ; Find the probability distribution of  $X$ .
15. A random variable  $X$  assumes 7 values  $-3, -2, -1, 0, 1, 2, 3$  with equal probability, find  $E(X)$  and  $E(X^2)$ .
16. Show that  $\text{Var}(aX) = a^2 \text{Var}(X)$ , where  $a$  and  $b$  are constants.
17. With usual notations, prove that  $E(X-k)^2 = \text{Var}(X) + [E(X)-k]^2$
18. Prove that  $E(X^2) \geq [E(X)]^2$
19. For a discrete random variable  $X$ ,  $E(X) = 10$  and  $\text{Var}(X) = 25$ . Find the positive values of  $a$  and  $b$  such that  $Y = aX-b$  has mean 0 and variance 1.
20. Show that  $E(aX + b) = aE(X) + b$
21. A random variable  $X$  has following probability mass function

$X$	- 2	-1	0	1	2	3
$P(X=x)$	0.1	$k$	0.2	$2k$	0.3	$k$

Find value of  $k$  and calculate mean and variance of  $X$ .

22. Let  $X$  be a random variable with following as the p.m.f. :-

$X$	0	1	2	3
$P(X=x)$	0.1	0.3	0.4	0.2

Find  $E(X)$  and  $\text{Var}(X)$ .

23. 17. Let  $X$  be discrete random variable with p.m.f.

$$P(X = x) = \begin{cases} \frac{x}{15} & ; \text{for } x = 1, 2, 3, 4, 5 \\ 0 & ; \text{otherwise} \end{cases}$$

Find  $E(X)$  &  $\text{Var}(2X-3)$ .

24. A roulette wheel is divided into 25 sectors of equal area numbered from 1 to 25. Let  $X$  be the number that occurs when the wheel is spun. Find:
- A) Probability mass function (p.m.f.) of  $X$
- B)  $P(X \geq 15)$
- C) Mean of  $X$
- D) Standard Deviation of  $X$

**D) Questions for 6 mark:-**

1. A weighted coin [  $P(H) = \frac{2}{3}$ ,  $P(T) = \frac{1}{3}$  ] is tossed three times .If the variable X denotes the number of heads produced in three tosses, find

- i) the p.m.f of X
- ii) the c.d.f of X
- iii)  $P[X=2]$

**CHAPTER 4: STANDARD DISCRETE DISTRIBUTION**

**A) Questions for 1 mark:-**

**I] Choose the correct alternative**

1. Suppose X and Y are two independent discrete uniform random variables with parameter n. The distribution of X and Y is
  - a) Discrete uniform Distribution with parameter 2n
  - b) Binomial distribution with parameter 2n
  - c) Discrete uniform Distribution with parameter  $n^2$
  - d) Poisson distribution with parameter 2n
2. If X follows discrete uniform distribution on  $0,1,\dots,n$  and the mean of the distribution is 6 then the value of n is
  - a) 6
  - b) 18
  - c) 36
  - d) 12
3. If random variable X has binomial distribution with parameters n and p, then
  - a) Mean < variance
  - b) Mean > variance
  - c) Mean = variance
  - d) Mean  $\leq$  variance
4. If  $X \sim B(n,p)$  and  $E(X) = 5/3$ ,  $\text{var}(X) = 10/9$ , then the value of p is
  - a)  $1/3$
  - b)  $2/3$
  - c)  $1/6$
  - d)  $5/6$
5. If  $X \sim B(n_1,p)$ ,  $Y \sim b(n_2,p)$  and X and Y are independent, then the distribution of  $X+Y$  is
  - a)  $B(n, 1/4)$
  - b)  $B(4n, 1)$
  - c)  $B(n, 3/4)$
  - d)  $B(2n, 1/4)$

6. Let  $X \sim P(\lambda=2)$ . Variance of  $X$  is:

- a) 4
- b) 9
- c) 2
- d) 1.4142

**II] State whether each of the following statement is true or false.**

- 1. Mean and standard deviation of discrete uniform distribution are equal.
- 2. Binomial distribution is negatively skewed.
- 3. A Bernoulli trial is a random experiment which has only two outcomes.
- 4. Mean of Bernoulli distribution is less than its variance.
- 5. Binomial distribution is particular case of Bernoulli distribution.
- 6. In Binomial distribution, mean is greater than variance.

**B) Questions for 2 marks:-**

- 1. Give two real life situations where uniform distribution can be realized.  
Find the mean of uniform distribution.
- 2. State which distribution can be applicable in following situation
  - a) The birth day of a parson it may be either Sunday, Monday - - - Saturday.
  - b) A computer generates a digit randomly.
- 3. The p.m.f. of uniform random variable  $X$  is  
 $P(x) = k$  for  $X=5,6,7,\dots,16$ ,  $k>0$ . Find the value of  $k$ .
- 4. Obtain mean and variance of discrete uniform variable taking values  $1,2,\dots,n$ .
- 5. Define Bernoulli distribution. Give one examples of real life situations where Bernoulli distribution can be used.
- 6. Define Binomial distribution. Find mean of binomial distribution.
- 7. For Binomial Distribution, Is it possible to have  $E(X) = 3$  and  $V(X) = 5$ ? Justify
- 8. State the p.m.f. of Poisson distribution.
- 9. If  $X \sim B(n, p)$  then state distribution of  $Y = n - X$ ?

**C) Questions for 4 marks:-**

1. Define discrete uniform probability distribution & obtain its mean and variance.
2. Obtain mean and Variance of Bernoulli distribution.
3. State and prove additive property of Binomial distribution.
4. Find mean and variance of Poisson distribution.
5. State and prove additive property of Poisson distribution.
6. Prove that under certain conditions to be stated, Binomial distribution is approximated by Poisson distribution.
7. Suppose  $X \sim B(n, p)$  if  $E(X) = 6$  &  $\text{Var}(X) = 4.2$  find  $n$  and  $p$ . Also compute  $P(X=2)$
8. Suppose  $X \sim B(n, p)$  if  $p = 0.6$   $E(X) = 6$ , find  $n$  and  $\text{Var}(X)$
9. Let  $X \sim B(n=8, p=1/4)$  find i)  $P(X=3)$  ii)  $P(X \leq 3)$  iii) Mean and Variance of  $X$
10. A fair coin is tossed 10 times. Find the probability of getting  
i) Exactly 6 heads      ii) at least 2 heads      iii) at most 8 heads
11. Explain how binomial distribution is applied in case of SRSWR.
12. State and prove the recurrence relation for binomial probability? What is its use?
13. Let  $X$  follows Binomial distribution with mean = 10 and Variance = 5.  
Find i)  $P(X \leq 5)$     ii)  $P(2 \leq X \leq 10)$       iii)  $P(X \leq 10)$
14. For a  $B(n, p)$  distribution,  $P(X=1) = 0.0768$ ,  $P(X=2) = 0.2304$
15. Define Binomial distribution. State relation between Bernoulli random variable and Binomial random variable.
16. Let  $X$  be a discrete random variables. Define  $E(X)$  and  $E(X^2)$ . Hence give formula for variance of  $X$ .
17. Number of cars hired by a car hire company on a day follows Poisson distribution with mean 3. Find the probability that  
on a given day :  
i) No car is hired  
ii) More than 3 cars are hired.

**D) Questions for 6 marks:-**

1. The p.m.f. of uniform random variable  $X$  is  $p(x) = k$  for  $x=5,6,7,\dots,16$ ,  $k>0$   
Find  
i)  $k$   
ii)  $P(X \geq 8)$   
iii)  $P(8 \leq X \leq 14)$

iv)  $(P(X \geq 10))$

2. A special die has  $(n+1)$  faces marked with numbers  $0,1,2,\dots,n$ . Assuming that the die is unbiased, find the probability distribution of the number on the top face. , find also the probability that the number on the top face is greater than 4 but less than or equal to  $(n-1)$
3. Obtain mean and Variance for a Bernoulli random variable. When is the distribution symmetric?
4. Show that product of  $n$  independent Bernoulli ( $p$ ) random variables is a Bernoulli random variable with parameter  $p^n$ .
5. Define Binomial distribution. Give interpretation of a  $B(n, p)$  random variable. Also Mention three real life situations, where the distribution's application.
6. Let  $X \sim B(n, p)$ . Obtain the mean and Variance of  $X$ . show that  $\text{Var}(X) < E(X)$
7. A student prepares for an examination by studying a set of 10 problems. He can work 7 of the 10. if the professor chooses randomly 5 of the 10 problems for the exam. What is the probability that the student can work at least 4 of them?
8. If the probability that any person 65 years old will be dead within a year is 0.05 Find the probability that out of a group of 7 such persons
  - i) Exactly one
  - ii) None
  - iii) at least one
  - iv) Not more than one
  - v) all of them will die within a year.
9. Let  $X$  &  $Y$  be two independent binomial variation with parameters  $(n_1=5, p=0.4)$  &  $(n_2= 6, p = 0.4)$  respectively.  
Find
  - i)  $P(X+Y = 4)$
  - ii)  $P(X+Y \leq 8)$
  - iii)  $P\{X= 1/(X+Y= 7)\}$
  - iv)  $P\{Y= 2 / (X+Y = 8)\}$
10. Let  $X \sim \text{Poisson}(\alpha)$  and  $Y \sim \text{Poisson}(\beta)$  be two independent random variables. Define a new random variable as  $Z= X+Y$ . Find the PMF of  $Z$ .
11. You take an exam that contains 20 multiple choice questions .Each question has 4 possible options. You know the answer to 10 questions, but you have no idea about the other 10 questions so you choose answers randomly. Your score  $X$  on exam is the total number of correct answers. Find the PMF of  $X$ . What is  $P(X>15)$ ?

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