Anekant Education Society's

Tuljaram Chaturchand College of

Arts, Science and Commerce, Baramati

(Autonomous)

QUESTION BANK

FOR

F.Y.B.Sc SEM-I

STATISTICS

PAPER-II: STAT-1102

DISCRETE PROBABILITY AND PROBABILITY DISTRIBUTIONS-I

(With effect from June 2019)

Paper – II (STAT 1102)

Discrete Probability and Probability Distributions-I

Unit-1Sample Space and Events

A) Questions for 1 mark

I] Choose the correct alternative

1. Relative Complement of A w. r. t. B is given by

   a) $A \cap B'$ b) $A' \cup B$ c) $A' \cap B$ d) $(A \cap B)'$

2. In an experiment of planting four seeds, the number of seeds germinated after a week are recorded. The Sample space of this eXperiment is

   a) $(0,4)$ b) $\{1,2,3,4\}$ c) $[0\ 4]$ d) $\{0,1,2,3,4\}$

3. If $A \cap B = \phi$ then the two events A and B are

   a) Mutually exclusive b) Exhaustive events

   c) Independent events d) Dependent events

4. If $A \cap B = \Omega$ then the two events A and B are

   a) Mutually exclusive b) Exhaustive events

   c) Independent events d) Dependent events

5. Which of the following is a pair of mutually exclusive events in drawing of a single card from a pack of 52 playing cards?

   a) a heart and a queen b) an even number and a spade

   c) an ace and an odd number d) a club and a red card

6. Consider an experiment of rolling a die. Suppose A is the event that an odd number appears and B is the event that a prime number appears. Which of the following statements is true?

   a) A and B are mutually exclusive b) $A \subset B$

   c) A and B are not mutually exclusive d) $B \subset A$

7. If an unbiased coin is tossed once, then the two events 'getting head' and 'getting tail' are

   a) Equally likely events b) Exhaustive events

   c) Mutually exclusive events d) All a), b), c) are true.

II] State whether each of the following statement is True or False.

1. A and B are mutually exclusive events if and only if $A \cap B = \phi$.
2. A and B are exhaustive events if and only if $A \cup B = \Omega$.
3. The event Ω and ϕ are mutually exclusive.
4. A ball is thrown in the air it will fall down is the example of deterministic experiment.
5. It is not possible to define impossible event.
6. A discrete sample space must contain a finite number of elements.
7. Occurrence of at least one of the two events is the event corresponding to the union of the two events.
8. Simultaneous occurrence of the two events is the event corresponding to the intersection of the two events.
9. An elementary event is an event containing only one element.

A) Questions for 2 marks

1. Define random experiment.
2. Classify the following experiments into deterministic and Non – deterministic experiment.
   i) The agriculturist uses different types of fertilizers to see which maximizes the yield of a crop.
   ii) Marketing manager conducts market survey to measure the effect of advertisement on sales.
   iii) Water is heated up to 100^0c.
3. Define sample space and give one examples of it.
4. Give two examples of countably infinite sample space.
5. Give one example of sure event and impossible event.
6. Write down sample space for the following experiment. Also mention its type.
   i) Answers to an objective question which has four multiple choices A, B, C, D. Student ticks a single answer.
   ii) Number of tossing of a die till 6 appear for the first time.
   iii) A two digit number is formed from the digits 4, 5, 6 using each digit only once.
   iv) A point is randomly placed inside a circle with radius 10cm. Its distance from the center is measured.
7. Give two examples of countably infinite sample space.
8. Give one example of sure event and impossible event.
9. Define an elementary event
10. Define compound event
11. Define mutually exclusive events.
12. Define exhaustive events.
13. What do you understand by equally likely events?

14. A coin is tossed till head occurs for the first time. Write down the sample space of this experiment.

C) Questions for 4 Marks

1. Define sample space. Explain types of sample spaces giving examples of each.
2. What do you mean by an experiment? What are different types of experiments? Explain.
3. Define with example each of the following

   i) Sample space ii) Discrete sample space iii) Event iv) Elementary event
4. Explain the concept of mutually exclusive and exhaustive events.
5. Write down the sample spaces for following experiment. Also state the type of the sample spaces.
   i) A sample is taken to predict the result of the particular assembly poll.
   ii) A three digit number is formed from the 5 distinct numbers, using each digit only once.
6. Distinguish clearly between deterministic and non- deterministic experiments. Give suitable examples.
7. Give an example of a random phenomenon that would be studied by
   i) a sociologist
   ii) an economist
   iii) a quality control engineer
   iv) physicist

D) Questions for 6 Marks

1. Explain with one illustration
   i) Complement of an event
   ii) Mutually exclusive events
   iii) Exhaustive events
2. Suppose three fair coins are tossed simultaneously. Let A be the event that exactly 2 coins show heads and B be the events that at least 2 coins show heads. List the elements of A, B, A' and B'. Verify whether A and B are i) mutually exclusive ii) exhaustive events.
3. Two fare dice are rolled. Let A be the event that sum of the points on the uppermost faces is even and B be the event that the product of the two numbers is not greater than 9. List the elements contained in the events- i) A U B ii) $A \cap B$ III) $(A \cap B') \cup A'$

Unit-2. Probability

A) Questions for 1 mark

I] Choose the correct alternative

1. In the simultaneous tossing of two fair coins, the probability of having one head is

   a) 0.5   b) 0.25   c) 0.75   d) 1

2. In the simultaneous tossing of two fair coins, the probability of having at least one head is

   a) 0.5   b) 0.25   c) 0.75   d) 1

3. Let A be event defined on sample space, Ω. Which of the following statement is true?

   a) P (A) = 1   b) P (A) = 0   c) $0 \le P(A) \le 1$ d) $0 \le P(A) < 1$

4. If $A \cap B = \phi$ then the two events A and B are

   a) Mutually exclusive   b) Exhaustive events

   c) Independent events   d) Dependent events

5. Suppose $\Omega = \{w_1, w_2, w_2, w_4,\}; P(w_1) = \frac{1}{7}, = P(w_2), P(w_3) = k, P(w_4) = \frac{3}{7}$ for what value of k will this be a probability model?

   a) 0   b) 1   c) 3/7   d) 2/7

5. If A and B are two events defined on Ω such that $A \subset B$ then

   a) P(A) = P(B)   b) P(A) < P(B)   c) $P(A) \le P(B)$ d) P(A) > P(B)

6. If P(A) = P(B) then the two events are

   a) Equally likely
   b) Dependent
   c) Independent
   d) Both a and b

7. When a card is drawn from the standard pack of playing cards, which of the following is a pair of disjoint events?

   e) An ace and an odd denomination
   f) A heart and a queen
   g) A club and a red card
   h) An even denomination and spade

8. The probability of an impossible event is

   a) 1   b) 1-P(ϕ)   c) P(Ω)   d) 0

9. If $A \subset B$, then the relation between P(A) and P(B) is :

   a) $P(A) \ne P(B)$ b) P(A) = P(B)   c) P(A) > P(B)   d) $P(A) \le P(B)$

10. If P(A) = 0.8, P(B) = 0.7, $P(A \cup B)$ = 0.96 then $P(A \cap B)$ is

    a) 0.56   b) 0.06   c) 0.6   d) 0

11. The probability of drawing one red ball randomly from a bag containing 5 red, 7 black and 10 yellow balls is

    a) 0   b) 1/5   c) 5/22   d) 1/22

12. If A and B are independent events with P(A) = 0.4 and P(B) = 0.25 then P (AUB) is

    a) 0.55   b) 0.65   c) 0.1   d) 0

13. If A and B are independent events with P(A) = 0.4, P(B) = 0.5 then $P(A' \cap B)$

    a) 0.03   b) 0.9   c) 0.1   d) 0.3

14. If $P(A \cap B) = 0.24$ and $P(A' \cap B) = 0.32$, then P(B) is

    a) 0.47   b) 0.56   c) 0.53   d) 0.51

II] State whether each of the following statement is True or False.

1. P (Ω) =1 is one of the axioms of probability.
2. Probability of drawing a card of king from 52 playing cards is 1/4.
3. In tossing a fare coin twice, Probability of getting two heads is 1/2.
4. P (∅) =0 is one of the axioms of probability.
5. Probability that the person will not die is 1.
6. We cannot use the classical definition of probability in case the sample space is countable infinite.

B) Questions for 2 marks

1. Give the classical definition of probability.
2. Define Equiprobable sample space.
3. Check whether following can be taken as probability models or not Justify.
    i) $P(w_1) = P(w_2) = P(w_3) = 1/3$, $P(w_4) = P(w_5) = P(w_6) = 1/6$
    ii) $P(w_i) = 1/8$ for all i=1,2,3,4,5,6
4. State axioms of probability.
5. Show that, for any event A, P(A')=1-P(A)
6. State and prove Booles' Inequality for two events.
7. Prove that the probability of an impossible event is zero.
8. Two cards are drawn from a well shuffled pack of 52 playing cards. Find probability that both cards are of diamond.
8. Four cards are drawn from a well shuffled pack of 52 playing cards. Find the probability that each card is of different suit.
9. Suppose a pair of fair dice is thrown. Find the probability that both the faces are same.
10. Suppose a pair of fair coin is tossed. Find the probability that at least one coin shows head.

11. Explain what is meant by i) Odds in favour of an event are a:b.

ii) Odds in favour of an event are a:b.

C) Questions for **4 marks**

1. Give Classical Definition of Probability. What are the limitations of classical probability?
2. Define i) Equiprobable sample space ii) probability of an event
3. A and B are two events defined on Ω such that A ⊆ B, then show that P(A) ≤ P(B)
4. State and prove addition theorem of probability for two events.
5. A factory employs both male and female workers. The probability that a male worker chosen at random is 0.65 and probability that the worker is married is 0.7 and that the worker is a married male is 0.47 . Find the probability that a worker chosen at random is

   1) a married female and 2) a male or married or both
6. Consider the experiment of rolling two fair dice. Let A= odd number on the first die , B= odd number on the second die, and C= sum of two points is odd. Check whether A, B, and C are pairwise independent and mutually independent.
7. Consider a pack of playing cards without face cards. 4 cards from this pack are drawn at random. Find the probability that they belong to

   i) different suits ii) 2 are face cards
8. Define independence of two events A and B defined on a sample spaceΩ. Can two mutually exclusive events be independent? Justify with an illustration.
9. Let A and B be two events defined on the sample space Ω, such that P(A)=0.65 , P(B)=0.7 , P(A'UB')=0.53, then find i) P(A∪B) , ii) P(A'∩B), iii) P(A∩B') and iV) P(A'∩B').
10. In a random arrangement of the letters of the word "STATISTICS", find the probability of, all vowels occupy even places.
11. In a random arrangement of the letters of the word "INFINITY", find the probability of, all vowels come together.
12. How many 3 digits numbers can be formed from the six digits 1, 3, 5, 6, 7, 9 . If each digit is to be used only once. Among these how many will be divisible by 5.
13. An integer is chosen at random from 1 to 100. What is the probability that the number is:-
    i) divisible by 5
    ii) not divisible by 7
14. Ten students are seated at random in a row for a photograph. Find the probability that two particular students are not seated side by side.

15. A die is loaded so that probability of an even number is twice the probability of an odd number. Even numbers are equally likely. Find the probability that

i) a prime number appears on the uppermost face

ii) an even number appears

iii) an odd number appears

iv) perfect square appears

16. If A, B, C are three mutually exclusive and exhaustive events defined on sample space Ω. If 3P(A) = 2P(B) = P(C). Find P(AUB).

17. Find the probability of the event A if the odds in its favour are 3:2.

18. Find the probability of the event A if the odds in its against are 1:4.

D) Questions for **6 marks**

1. For any two events A and B defined on Ω, show that P (A U B) ≤ P(A) + P(B)
2. State and prove addition theorem of probability for three events.
3. If A_1, A_2…A_n are events defined on Ω then show that P(∪A_i) ≤ Σ P(A_i)
4. Let A, B, C be any three events defined on Ω, such that P(A) = 3/8, P(B) = P(C) = 1/4 P(B ∩ C) = 0, P(A∩B)=1/8=P(A∩C) Evaluate i) P(A U B U C) ii) P(A U C) iii) P(A’ ∩B’ ∩ C’)
5. Which of the following events has the greatest probability? Justify.

   i) getting 4 on throwing a single die

   ii) getting a sum of 8 on throwing two dice

   iii) getting a sum of 12 on throwing three dice
6. A husband and wife appear for two vacancies in the same post. The probability of husband’s selection is 1/7 and that of wife’s selection is 1/5. What is the probability that i) both of them will be selected, ii) only one of them will be selected, iii) none of them will be selected.

Unit-3 . Conditional Probability and Independence of events:

A) Questions for 2 marks

I] Chose the correct alternative

1. In conditional probability distribution of Y given X=x,

   a. X is variable b. X is constant c. Y is variable d. Y is constant.

2. Which of the following statement is true?

   a. A and A' form partition of Ω
   b. A and Ω form partition of Ω
   c. A and A' do not form partition of Ω
   d. Only two events cannot form partition of Ω

3. If A is an event defined on Ω then P(A | A') is ,

   a. 1 b. P (A) c. 0 d. P (A')

4. If A is an event then conditional probability of A given that A has already occurred is

   a. 1 b. 0 c. 0.5 d. 0.75

5. Which of the following statement is true?

   a. $P(A/B) \geq P(A)$ b. $P(A/B) \leq P(A)$

   c. $P(A/B) = P(A)$ d. nothing can be said about magnitudes of P(A) and P (A/B) .

6. If A and B are independent events with P(A) =0.4 and P(B) = 0.5 then $P(A \cap B)$ is

   a. 0.1 b. 0 c. 0.2 d. 0.02

II] State whether each of the following statement is True or False.

1. If $A \subset B$ then P(A | B)=1
2. Independence implies mutual exclusiveness.
3. If A and B are independent then A' and B' are also independent events.
4. Bayes' theorem is used to calculate posterior probabilities of events.

C) Questions for 2 marks

1. For events A and B if P(A) = P (A/B) = 0.15 and P (B/A) = 0.20 , then check whether A and B are independent.
2. State Bayes' theorem.
3. Define independence of two events A and B on Ω.
4. Describe when two events defined on sample space Ω are independent as well as mutually exclusive?

5. A and B are two events defined on a sample space Ω state the nature of relationship between A and B if (a) P (A/B) = 0 and (b) P (A/B) = P (A).

6. If A and B are independent events with P(A) =0.4 and P(B) = 0.5 then P(A'∩B').

C]. **Questions for 4 marks**

1. Define independence of two events A and B on Ω. Give an illustration.
2. Does independence of two events imply that the events are mutually exclusive? Justify.
3. Does mutually exclusiveness of two events imply independence? Justify.
4. Given that A, B and C are three events defined on Ω then define
   (i) Pair wise independence (ii) mutual independence of A, B and C.
5. Prove or disprove that
   (i) Mutual independence of 3 events ⇒ pair wise independence
   (ii) Pair wise independence ⇒ mutual independence
6. State and prove multiplication theorem for two events A and B defined on a sample space Ω.
7. For two events A and B defined on sample space Ω , prove or disprove P (A/B) = P(B/A) .
8. A number is selected at random from 1, 2, 3, 4, 5, 6, 7, 8. Define A={1,2,3,4}, B={2,4,6,8}, C={1,3,5,7} then check whether A, B and C are i) pair wise independent ii) mutual independent..
9. Let A_1, A_2, A_3 completely independent events defined on Ω. If P (A_i) = $(1/2)^i$ i = 1 , 2, 3 find
   P ($\bigcup_{i=1}^{n} A_i$)
10. A, B and C form a partition of the sample space Ω . If 3P(A)=2P(B)=(C) then find P(AUB).
11. There are 4 boys and 2 girls in Room No.1 and 5 boys and 3 girls in Room No.2. A girl from one of the rooms laughed loudly. What is the probability that the girl who laughed loudly was from Room No.2?
12. The probability that a man will be alive 20 years hence is 0.3 and probability that his wife will be alive 20 years hence is 0.4. Find the probability that after 20 years.
    (i) Both will be alive
    (ii) Only the man will be alive
    (iii) Only the wife will be alive and
    (iv) At least one of them will be alive.

12. An article manufactured by a company consists of two parts A and B. In the manufacturing process of part A, 9 out of 100 are likely to be defective; similarly 5 out of 100 are likely to be defective in the process of part B. Calculate the probability that the assembled parts will not be defective.

13. Bag I contains 6 blue and 4 red balls. Bag II contains 2 blue and 6 red balls. Bag III contains 1 blue and 8 red balls.
    i) A bag is chosen at random; a ball is drawn randomly from this bag. It turns out to be blue. Find the probability that bag I was chosen.

ii) A bag is chosen at random; two balls is drawn randomly from this bag without replacement from this bag. Both the balls were blue. Find the probability that bag II was chosen.

14. An explosion in a factory manufacturing explosives can occur due to (i) short circuit (ii) defects in machinery (iii) negligence of workers (iv) sabotage. The probabilities of these four causes are known to be 0.3, 0.2, 0.4 and 0.1 respectively. The engineers feel that an explosion can occur with probability (i)0.3 if there is a short circuit,(ii) 0.2 if there are defects in machinery,(iii) 0.25 if the workers negligent and (iv)0.8 if there is a sabotage .Given that an explosion has occurred , determine the most likely cause of it.
15. Consider the experiment of rolling two fair dice. Let A= odd number on the first die, B= odd number on the second die, and C= sum of two points is odd. Check whether A, B, and C are pairwise independent and mutually independent
16. In a certain school, examination results showed that 20% students failed in Mathematics, 5% failed in English while 10% failed in both Mathematics and English. Are the two events failing in Mathematics and failing in English independent. ?
17. A town has 3 doctors A, B and C operating independently. Then probability that doctor A available is 0.9 and that for doctor B is 0.6, for doctor C is 0.7; what is the probability that at least one doctor is available when needed ?
18. There are two sections I and II in Statistics paper. The probability that a candidate passes in Section – I is 0.6 and that he passes in Section-II is 0.7. What is the probability that a particular candidate passes only in any one of the two sections?
19. Two urns identical in appearance contain respectively 3 white and 2 black balls, and 2 white and 5 black-balls. One urn is selected at random and a ball is drawn from it. What is the probability that it is black?
20. If A and B are two events on a sample space Ω. Prove with usual notation.
    P (A) = P (A/B). P (B) + P (A/B') P (B')
21. If A and B are mutually exclusive non empty events of Ω then show that
    P (A/AUB) = P (A) / (P (A) + P (B))
22. Suppose A1, A2, A3 form partition of the sample space Ω with P(A1) = P(A2) = P (A3) = 1/3 and P(B/A1)=2/7, P(B/A2) = 4/9, P (B/A3) = 1/5, find P (A1/B).

D] Questions for 6 marks

1. If A and B are two independent events defined on Ω; then prove that.
   (a) A and B' are independent
   (b) A' and B are independent
   (c) A' and B' are independent
2. Show that conditional probability satisfies all axioms of probability.

3. If A, B, C are any three events defined on Ω with P (B) > 0 , then prove that P (AUC/B) = P (A/B) + P (C/B) – P(A∩C/B)
4. Define partition of a sample space. State and prove Baye's theorem.
5. If A_1, A_2------ A_n are partition of a sample space and if B is any event defined on Ω , prove that P(B) = $\sum_{i=1}^{n} P(A_i)P(B/A_i)$
6. Let A, B and C be three events defined on sample space Ω where A and B are mutually exclusive, A and C are independent. B and C are independent. If P(A) = 1/4, P(B) = 1/3, P(C) = 1/6. Find i) P(AUB) (ii) P (A∩C) (iii) P (AUBUC).
7. A husband and wife appear for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that
   i) Both of them will be selected?
   ii) Only one of them will be selected?
   iii) None of them will be selected?
8. In a bolt factory, three machines A, B, C produce respectively 25%, 35% and 40% of the days production. Out of the total output 5%, 4% and2% are defective bolts respectively from A, B and C. A single bolt is drawn at random from the days' production and is found defective. Determine the probabilities that it was manufactured by machines A, B and C.
9. Bag I contains 3 blue and 4 red balls. Bag II contains 2 blue and 6 red balls. Bag III contains 1 blue and 8 red balls. A bag is chosen at random; a ball is drawn randomly from the bag. It turns out to be blue. Find the probability that bag – I was chosen.
10. Consider that there are three identical bags, A, B and C. The bag A contains 2 gold coins bag B contains 2 silver coins and bag C contains 1 silver and 1 gold coin. What is the probability of selecting bag A out of the three given that a gold coin is selected?
11. Two boxes B1 and B2 containing marbles are placed on a table.B1 contains 7 green and 4 white marbles. B2 contains 3 green and 10 yellow marbles. The boxes are arranged so that the probability of selecting box B1 is 1/3 and the probability of selecting box B2 is 2/3. Sara is blindfolded and asked to select a marble. She will win LED T.V. if she selects a green marble.
    i) What is the probability that Sara will win the T.V.?
    ii) If Sara wins the T.V. what is the probability that the green marble was selected from the box B1.

Unit-4. Univariate Probability Distributions (finite sample space)

A) Questions for 1 mark

I] Choose the correct alternative

i) If X is a discrete random variable with c.d.f. F(X). Let a and b be two real numbers such that a<b then

a) $F(a) < F(b)$ b) $F(a) \le F(b)$ c) $F(b) = F(a)$ d) $F(a) > F(b)$

ii) If X and Y denotes the points obtained when two six faces unbiased dice are thrown ,then P(X=Y) is

a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{1}{24}$ d) $\frac{1}{36}$

iii) Which of the following is not a discrete random variable?

a) Number of students present in the class
b) Number of persons possessing 'O-ve' blood group in a blood camp donation
c) Number of daughters born to a couple until they get son
d) Weight of a new born baby.

iv) Median M of a discrete random variable X is that value for which

a) $P(X \le M) \ge \frac{1}{2}$ and $P(X \ge M) \ge \frac{1}{2}$

b) $P(X \le M) < \frac{1}{2}$ and $P(X \ge M) \ge \frac{1}{2}$

c) $P(X \le M) \ge \frac{1}{2}$ and $P(X \ge M) < \frac{1}{2}$

d) $P(X \le M) < \frac{1}{2}$ and $P(X \ge M) < \frac{1}{2}$

v) Let X be a discrete random variable with following p.m.f.

X	-3	-2	-1	0	1
P(X)	0.15	0.3	0.35	0.15	0.05

then Mode of X is

a) 0 b) 1 c) -1 d) -3

vi) Let X be a discrete random variable with following c.d.f.

X	-1	0	1	2	3
F(X)	0.15	0.45	0.8	0.95	1

then median of X is

a) 0 b) 1 c) -1 d) 3

vii) Suppose we toss a biased coin twice. Probability of getting 'head' is twice of getting 'tail' for the coin then probability of getting no head is

a) 0 b) $\frac{1}{3}$ c) $\frac{1}{9}$ d) $\frac{4}{9}$

II] State whether each of the following statement is True or False.

1. A discrete random variable assumes only finite number of values.
2. A discrete. r. v. is defined on a discrete sample space.
3. Sum of probabilities of all values of a random variable is less than one.
4. The p.m.f .of a discrete .random variable is a non decreasing function.
5. A distribution function is an increasing function.

III] Define the following

1. Discrete random variable
2. Probability mass function of discrete random variable
3. Probability distribution of discrete random variable
4. Cumulative distribution function of a discrete random variable
5. Define mode of discrete probability distribution.
6. Define median of discrete probability distribution.

B) Questions for 4 marks

1. Verify whether the following can be looked upon as p.m.f. for the given values of X.

   i) P(X)=1/4 X=0,1,2,3.

   ii) P(X)=(X+1)/10 X=0,1,2,3.

   iii) P(X)=(X-2)/5 X=1,2,3,4,5.

   iv) P(X) = X2 /14 X=1,2,3.

2. Define cumulative distribution function (c.d.f.) of a discrete r.v. and state its properties.

3. The probability distribution of a discrete r.v. X is as follows:

X	0	1	2	3
P[X = x]	0.20	0.25	0.35	0.20

   i) Obtain cumulative distribution function of X.

   ii) Obtain median and mode of X.

4. Determine k such that the following functions are p.m.f

   i) f(x)=kx x=1,2,3,--------,10.

   ii) $f(x)=\frac{k\ 2x}{x!}$ x=0,1,2,3.

   iii) $f(x)=k(2x^2+3x+1)$, x=0,1,2,3.

5. The Probability distribution of a discrete random variable X is as follows:

X	0	1	2
P(X=x)	0.25	0.50	0.25

Find: i) $P(X\geq 0)$,ii) $P(X>1/X>0)$, iii)E(X).

6. The Probability mass function (p.m.f) of a r.v. X is given by

$P(X=x) = K\ {}^{5}C_{X}$; X=0,1,2,3,4,5.

; K>0

= 0 ; otherwise

Find the value of K.

7. Determine k such that the following function is p.m.f.

P(X=x) = k2X/x! ; X=0,1, 2, 3

= 0 ; otherwise

8. Given the following c.d.f. F(X) of discete .r.v. X obtain the p.m.f. of X also obtain the median and mode of X.

i) F(x) =0 X<0

=0.2 $0\leq X<2$

=0.5 $2\leq X<4$

=0.7 $4\leq X<6$

=0.8 $6\leq X<8$

=1 $X\geq 8$

ii) F(x)=0 X<1/4

=1/8 ¼≤ X<1/2

=1/4 ½≤X<3/4

=1/2 ¾≤X<1

=3/4 1≤X<5/4

=1 5/4≤X3/4

iii) F(x)=0 if X<5

=1/8 $5\leq X<10$

=7/24 $10\leq X<15$

=2/3 $15\leq X<20$

=11/12 $20\leq X<25$

=1 $X\geq 25$.

9. Let P(x) be p.m.f. of discrete r. v. X which assumes the values x1, x2, x3, x4 such that

$2P(x_1) = 3P(x_2) = P(x_3) = 5P(x_4)$

C) Questions for 6 marks

1. A discrete .r. v. X assumes values -2.5, -1.5, 0.5, 1.5 and 2.5 if P(X=-2.5)=P(X=-1.5),P(X=1.5)=P(X=2.5), P(X<0.5)=P(X>0.5)=P(X=0.5) Obtain (i) p.m.f. of X.(ii)the distribution function of X.
2. A r.v. X assumes values 1, 2, 3, 4, 5 such that P(X=1) =P(X=2); P(X=4) =P(X=5); and P(X<3) =P(X=3) =P(X>3) write down p.m.f. of X and evaluate P(X≤3).
3. A r.v. X takes values 0, 1, 2, 3, 4 such that P(1<X≤4) = 0.55 ; P(X≤1)=0.25; P(X=2)=2P(X=1)and P(0<X≤2)=0.45; Find probability distribution of X.
4. A d.r.v. X assumes values -3, -2, -1, 0, 1 , 2, 3 such that , P(X=-3)=P(X=-2)=P(X=-1) ; P(X=1)=P(X=2)=P(X=3); P(X<0)=P(X=0)=P(X>0); Obtain the probability distribution function and cumulative distribution function of X. Hence Find probability distribution function of Y=3X+4 . Also find median of Y.
5. A r.v. X has following probability distribution

X	0	1	2	3
P(x)	1/5	2/5	1/5	1/5

Find the Probability distribution of

i) W = X-1

ii) Y =(3X+2)/2

iii) Z = X^2+2

6. For the following distribution function F(X) of a discrete random variable X.

X	1	2	3	4	5	6	7	8
F(x)	0.08	0.12	0.23	0.37	0.48	0.62	0.85	1

Find

i) Probability distribution of X

ii) P(X≤4) and P(2≤X≤6)

iii) P(X=5/X≥3)

iv) P(X≥6/X≥4)

v) Median of the distribution.

7. Given the following C.d.f. of a discrete r. v. X:

X	1	2	3	4	5
F(X)	0.l0	0.26	0.52	0.78	l.0

Find:

i) Probability distribution of r. v. X

ii) P[X < 3].

iii) P[X > 3].

iv) P[X = 4 / X > 3].

8. Determine k such that the following function is a p.m.f.:

$P(X= x) = k (x^2 + 2x + 1)$, X : 0, 1, 2, 3

=0 , otherwise

Also find P(X = 1 / X < 2)

9. The p.m.f. of X is given by,

P(x) = k x ; X = l, 2, 3, 4, 5

= 0 ; otherwise

Find i) k ii) P(X < 3 | X is odd) iii) Median of X.

10. The cumulative distribution function (c.d.f) of a discrete random variable X is given below:

X	1	2	3	4	5	6
F(x)	0.15	0.35	0.45	0.68	0.86	1

Find: i) The probability distribution of random variable X.

ii) P(X=5 / X≥3)

iii) The values of mode and median of the distribution.

11. The probability distribution of a discrete random variable X is as follows:

X	-2	-1	0	1	2
P(x)	0.15	0.25	0.05	0.35	0.2

Find i) P (X > 0)

ii) P (-2 ≤ X ≤ 1)

iii) C.D.F. of X

iv) Median of X.

v) Mean and Variance of X

12. If a random variable X has following probability distribution function

X	-3	-2	-1	0	1
P(x)	0.1	0.2	0.25	0.3	0.15

Obtain i) $P(-2<X<1)$ ii) $P(|X| <1)$ iii) C.d.f. of X

Unit-5 Mathematical expectation (Univariate random variable)

A) Questions for 1 mark

I] Choose the correct alternative

1. If X is a discrete r.v., then

   a) $E(X^2) = [E(X)]2$ b) $E(X^2) > [E(X)]2$

   c) $E(X^2) < [E(X)]2$ d) $E(X^2) = 2E(X)$

2. If X is a degenerate random variable then:

   a) $E(X^2) < [E(X)]2$ b) $E(X^2) > [E(X)]2$

   c) $E(X^2) = [E(X)]2$ d) None of these

3. For a degenerate distribution at X = c, mean and variance are-

   a) (c , c) b) (c , 0) c) (0, 0) d) (0 , c)

4. If E(X) = 4 and Var (X) = 3, then Var (2X − l) is equal to

   a) 11 b) 12 c) 5 d) 7

5. Var (aX+b) =

   a) V(X) b) a^2 V(X) c) V(X)+b d) aV(X)+b

6. If X be a random Variable with mean 5 and variance 16. What are the values of mean and standard deviation of (X-5)/16?

   a) 0,1 b) 0,0.25 c) 5,1 d) 5,0.25

7. $M_X(0)$ is

   a) 0 b) 1 c) Constant d) ∞

8. If E(Y) = 3 where $Y = \frac{X-2}{5}$, then E(X) is

   a) 1/5 b) 17/5 c) 5/17 d) 17

9. If $\mu_1' = 3, \mu_2' = 11, \mu_3' = 25,$ then the value of γ_1 is

   a) 16.25 b) 21.43 c) 4.63 d) 14

II] State whether each of the following statement is True or False.

1 Variance of constant is zero

2 A discrete random variable cannot take negative values.

3 Variance of random variable is never negative.

4 The first raw moment of random variable is always zero.

5 If $\gamma 1 > 0$, then the distribution is positively skew.

6 If $\gamma 2 < 0$, then distribution is mesokurtic.

B) Questions for 2 marks

1. Define expectation of a r.v.
2. Define variance of a discrete random variable
3. Show that E(C)=C where C is constant.
4. Show that var(C)=0 where C is constant.
5. Define M.g.f.
6. Define p.g.f.
7. Show that $M_X(0) = 1$
8. If a discrete r. v. X has mean=4 and variance = 36 then find second raw moment of X.

C) Questions for 4 marks

1. Show that variance is invariant to change of origin but not of scale.
2. A random variable X assumes 7 values –3, -2, -1, 0, 1, 2, 3 with equal probability find E(X) and $E(X^2)$
3. Show that $Var(aX+b)=a^2$ Var(X), where a and b are constant
4. With usual notations, prove that $E(X-k)^2 = Var(X) + [E(X)-k]^2$
5. The probability mass function of a r.v. X is

$$P(x)=\frac{1}{10} \qquad ; x = 21, 22, 23, ...30$$
$$= 0 \qquad ; otherwise$$

   find E(X)
6. Prove that $E(X^2) \geq [E(X)]^2$
7. The probabilities that a man fishing at a particular place will catch 1, 2, 3, 4 fish are 0.4, 0.3, 0.15, 0.15 respectively. What is the expected number of fish caught?
8. Two cards are drawn at random from a box which contains five cards numbered 1, 1, 2, 2 and 3. Let X denotes the sum of the numbers. Find the expected value of the sum.
9. A baker man sells 5 types of cakes. Profit due to sale of each type of cake is respectively Rs. 1, 1.5, 0.5, 0.75 and 0.25. The demands for these cakes are 10%, 5%, 20%, 50% and 155 respectively. What is the expected profit per cake?
10. A discrete r.v. X has the following p.m.f;

$$P[X = x] = -\frac{x}{6} \qquad ; X=1, 2, 3$$
$$= 0 \qquad otherwise$$

   Find E[2X] and $E[X^2]$.

11. Let X be a discrete r.v. with p.m.f

$$P(X=X) = \frac{x}{15} \quad ; \text{for } X=1, 2, 3, 4, 5$$

$$= 0 \quad ; \text{otherwise}$$

Find i) E(X+5) ii) Var(X+5)

12. The probability distribution of a discrete r.v. X is a given below:

X	−2	0	l	2
P[X = X]	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

Find third central moment μ_3 . Also comment on the nature of the distribution.

13. The probability distribution of r.v.X is given by,

X	0	1	2	3
P[X = x]	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$

Calculate coefficient of skewness γ_1 and comment on the nature of the distribution.

14. For a discrete r.v. X, E(X) = 10 and Var(X) = 25. Find the positive values of a and b such that Y = aX-b has mean 0 and variance 1.
15. Show that E(aX+b) = aE(X)+b
16. If X and Y are two random variables then prove that E(X+Y) = E(X) + E(Y).
17. Explain how the raw moments can be obtained using m.g.f.
18. Show that m.g.f is affected by the change of origin and scale.
19. State and prove additive property of m.g.f.
20. If a disctrete r. v. X has m.g.f. $M_X(t) = e^{\frac{t^2}{2}}$ find first four raw moments of X.
21. If a disctrete r. v. X has m.g.f. $M_X(t) = e^{2t+3t^2}$ find first four cumulants.

D) Questions for 6 marks

1. A discrete random variable X take value 1,2,-----n with equal probability 1/n. find mean and variance of X if the ratio of var(X) to E(X) is equal to 4 find the value of n. What will be the value of n if var(X) = E(X)?.
2. If a disctrete r. v. X has m.g.f. $M_X(t) = e^{\frac{t^2}{2}}$ find first four raw moments of X and hence find first four central moments of X.
3. A r.v. X has following probability mass function

X	-2	-1	0	1	2	3
P(X=X)	0.1	k	0.2	2k	0.3	k

Find value of k and calculate mean and variance of X.

4. Let X be a r.v. with following as the p.m.f

X	0	1	2	3
P(X=X)	0.1	0.3	0.4	0.2

Find E(5-4X) and S.D.($\frac{X-3}{8}$).

5. Let X be discrete r.v. with p.m.f.

P(X=x) = $\frac{x}{15}$ X= 1, 2, 3, 4, 5

= 0 otherwise

Find E(X) and Var(2X-3).