

An Autonomous status

(Affiliated to Savitribai Phule Pune University, Pune)

DEPARTMENT OF PHYSICS

Classical Mechanics [PHY4102]

Question Bank for M.Sc students

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Unit-1: Constrained Motion and Lagrangian formulation

Objective Questions

1. The degree of freedom for a free particle in space are _____
(a) one (b) two
(c) **three** (d) zero
2. The number of independent variable for a free particle in space are _____
(a) zero (b) one
(c) two (d) **three**
3. The degree of freedom for N particles in space are _____
(a) 2N (b) **3N**
(c) N (d) zero
4. The number of independent variable for a free particle in space are _____
(a) N (b) 2N
(c) **3N** (d) zero
5. _____ constraints are independent of time.
(a) Holonomic (b) Non-Holonomic
(c) **Scleronomous** (d) Rheonomous
6. _____ constraints are time dependent.
(a) Holonomic (b) Non-Holonomic
(c) Scleronomous (d) **Rheonomous**
7. The Lagrangian equations of motion are _____ order differential equations.
(a) first (b) **second**
(c) zero (d) forth
8. The Lagrange's equations of motion for a system is equivalent to _____ equations of motion.
(a) **Newton's** (b) Laplace
(c) Poisson (d) Maxwell's

9. The Lagrangian function is define by _____

- (a) $L = F + V$ (b) $L = T - V$
(c) $L = T + V$ (d) $L = F - V$

11. The lagrangian for a charged particle in an electromagnetic field is

Where T is kinetic energy and ϕ and A are magnetic scalar and vector potentials

- (a) $L = T + q\phi + q(v \cdot A)$ (b) $L = T - q\phi - q(v \cdot A)$
(c) $L = T - q\phi + q(v \cdot A)$ (d) $L = T + q\phi - q(v \cdot A)$

12. The constraints on a bead on a uniformly rotating wire in a force free space is

- (a) **Rheonomous** (b) Scleronomous
(c) a and b both (d) None of these

13. Generalized coordinates

- (a) **Depends on each other** (b) Independent on each other
(c) necessarily spherical coordinates (d) May be Cartesian coordinate

14. If the lagrangian does not depend on time explicitly

- (a) **The Hamiltonian is constant** (b) The Hamiltonian cannot be constant
(c) The kinetic energy is constant (d) the potential energy is constant

15. Three particles moving in space so that the distance between any two of them always remain fixed have degree of freedom equal to

- a) 1 b) 3 **c) 6** d) 9

16. A non holonomic constrain may be expressed in the form of

- a) Equality **b) Inequality** c) Vector d) None of these

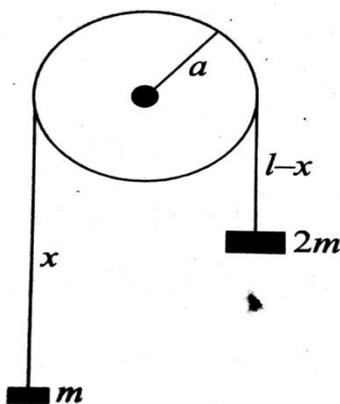
17. A particle is constrained to move along the inner surface of a hemisphere number of degrees of freedom of the particle

- a) 1 **b) 2** c) 3 d) 4

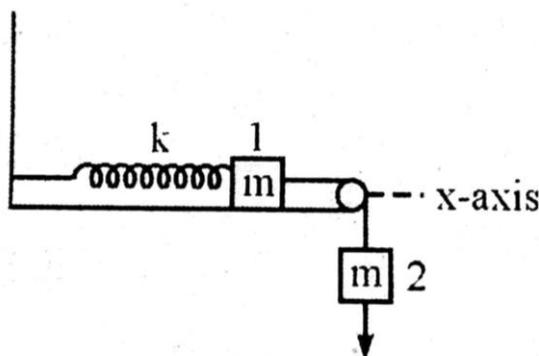
Short Answer Questions

1. Explain the meaning of Scleronomous and Rheonomous constraints. Give illustrations of each.
2. Discuss the concept of generalized coordinates with illustrations.
3. Explain the term 'virtual displacement' and state the principle of virtual work.
4. Solve the problem of Atwood machine by using D'Alembert's principle.
5. Discuss D'Alembert's principle.
6. Write the types of constraints for
 - 1) Motion of a body on an inclined plane under gravity
 - 2) A pendulum with variable length
7. Show that invariance of Lagrange's equation under Galilean transformation
8. Discuss the virtual work done for motion of a system and derive the mathematical statement of D'Alembert's statement.
9. Construct the Lagrangian of Atwood machine and derive its equation of motion.
10. Construct the Lagrangian of spherical pendulum and derive its equation of motion.
11. Find the Lagrangian and equation of motion for a bead slides on a wire with the shape of cycloid, described by equations $x = a(\theta - \sin\theta)$ & $y = a(1 + \cos\theta)$ where $0 \leq \theta \leq 2\pi$.
12. What are constraints? Discuss holonomic and Non-holonomic constraints with illustration.
13. "Simple pendulum with variable length" State constraint equation and classify the constraints.
14. Consider a particle moving in space. Using the spherical polar coordinates (r, θ, ϕ) as the generalized coordinates, express the virtual displacement $\delta x, \delta y$ and δz in terms of r, θ and ϕ .
15. Consider a particle moving in space. Using spherical polar coordinates (r, θ, ϕ) as the generalized coordinate express the virtual displacements $\delta x, \delta y$ and δz in terms of r, θ and ϕ .
16. What is compound pendulum? 1) Set up its Lagrangian 2) obtain its equation of motion 3) find the period of pendulum
17. To find the Lagrange's equation of motion for an electrical circuit comprising an inductance 'L' and capacitance 'C'. The condenser is charged to 'q' coulombs and the current flowing in the circuit is 'i' amperes.
18. Two heavy particles of weight W_1 and W_2 are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius R, the axis of which is horizontal. Find the condition of equilibrium of the system by applying principle of virtual work.

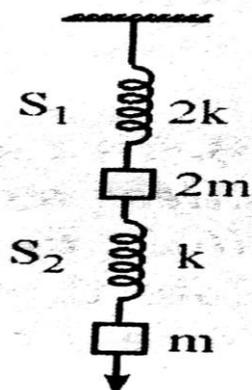
19. Consider the motion of a particle of mass m moving in space. Select the cylindrical coordinates (ρ, ϕ, z) as a generalized coordinates, calculate the generalized force components if force F acts on it.
20. Masses m and $2m$ are connected by a light inextensible string which passes over a pulley of mass $2m$ and radius a . Write the Lagrangian and find the acceleration of the system.



21. Two equal masses, joined by a rope passing over a light pulley are constrained to move on a frictionless surface. Find the expression for the extension of the spring as a function of time.

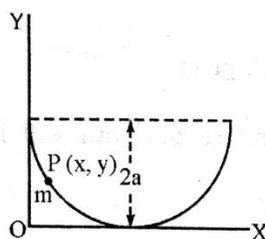


21. A mass $2m$ is suspended from a fixed support by a spring of spring constant $2k$. From this mass, another mass m is suspended by another spring of spring constant K . Find the equation of motion of the coupled system.
- 22.

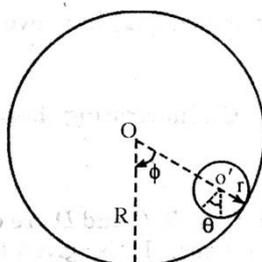


Long Answer Questions

1. What are constraints? Discuss various types of constraints with illustration
2. What are constraints? Write the types and equation of constraints for the following
 - I. Simple pendulum of variable length
 - II. Particle moving inside the box.
3. Derive the Lagrange's equation of motion for a conservative system from D'Alembert's principle.
4. Write the Lagrange's equation of motion for conservative system.
5. Write the Lagrange's equation of motion for non-conservative system.
6. What is compound pendulum? Obtain its equation of motion and find the period of pendulum.
7. What is Foucault's pendulum? Obtain its equation of motion and find the period of pendulum.
8. Compare Newtonian, Lagrangian and Hamiltonian formulation and discuss the advantages and disadvantages of each.
9. Is the Lagrangian formulation more advantageous than the Newtonian formulation? Why?
10. Show that invariance of Lagrange's equation under Galilean transformation.
11. A bead slides on a wire in the shape of a cycloid described by equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ where, $0 \leq \theta \leq 2\pi$. Find
 - I. The Lagrangian function
 - II. The equation of motion neglect friction between the bead and wire.

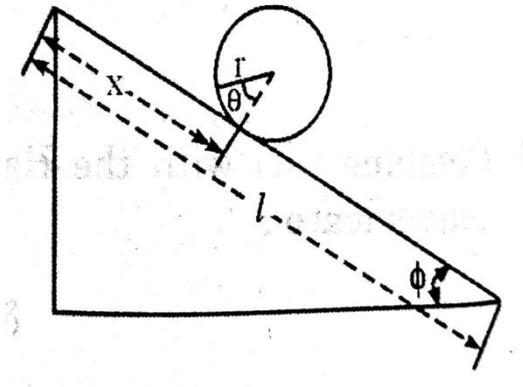


12. A solid homogeneous cylinder of radius r , rolls without slipping on the inside of a stationary large cylinder of radius R
 - a. Find the equation of motion
 - b. What is the period of small oscillations about the stable equilibrium position?



13. A hoop rolling down on inclined plane without slipping. Find the equation of constrain on the co ordinates x and θ

The coefficients are $a_\theta = r$ and $a_x = -1$.



Chapter-2: Hamilton's formulation & Variational Principle

Objectives

1. Generalized coordinates

- (a) **Depends on each other** (b) Independent on each other
 (c) necessarily spherical coordinates (d) May be Cartesian coordinate

2. If the lagrangian does not depend on time explicitly

- (a) **The Hamiltonian is constant** (b) The Hamiltonian cannot be constant
 (c) The kinetic energy is constant (d) the potential energy is constant

3. The lagrangian of a particle of mass m moving in a plane is given by

$$L = \frac{1}{2}m(v_x^2 + v_y^2) + a(xv_y - yv_x)$$

Where v_x and v_y are velocity components and a is a constant. The canonical momenta are given by

(a) $p_x = mv_x$ and $p_y = mv_y$ (b) $p_x = mv_x + ay$ and
 $p_y = mv_y + ax$

(c) $p_x = mv_x - ay$ and $p_y = mv_y + ax$ (d) $p_x = mv_x - ay$ and
 $p_y = mv_y - ax$

4. Hamiltonian canonical equation of motion for a conservative system are

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (b) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

5. A particle of charge q, mass m and linear momentum \vec{P} enters an electromagnetic field of vector potential \vec{A} and scalar potential ϕ . The Hamiltonian of the particle is

a) $\frac{P^2}{2m} + q\phi + \frac{A^2}{2m}$ b) $\frac{1}{2m} (\vec{P} - \frac{q}{c}\vec{A})^2 + 2\phi$

c) $\frac{1}{2m} (\vec{P} - \frac{q}{c}\vec{A})^2 + \vec{P} \cdot \vec{A}$ d) $\frac{P^2}{2m} \cdot q\phi - \vec{P} \cdot \vec{A}$

6. For the langrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ the Hamiltonian H is

a) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} = kxy$

b) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$

c) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$

d) $\frac{p_x^2 + p_y^2}{4ab} - kxy$

7. For the langrangian given by $L = \frac{m}{2}(\dot{r}^2 + r^2)\dot{\theta}^2 - \frac{v}{r}$. The generalized momenta are given by

a) $m\dot{r}$ and $mr^2\dot{\theta}$

b) $m\dot{r}$ and $mr\dot{\theta}$

c) $mr\dot{r}^2$ and $mr^2\dot{\theta}$

d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

Short Answer Questions

1. State and explain Hamilton's modified principle.
2. Distinguish between configuration space and phase space.
3. Lagrangian for one dimension harmonic oscillator is given by $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, Obtain corresponding Hamiltonian.
4. Explain how Hamilton's equation of motion can be expressed in terms of Poisson's bracket
5. Obtain Hamilton's equation for a simple pendulum. Hence, obtain an expression for its time period.
6. State and explain Hamilton's principle.
7. Distinguish between configuration space and phase space
8. Obtain Hamilton's equation for a simple pendulum. Hence, obtain an expression for its period.
9. Obtain Hamiltonian of a charged particle in an electromagnetic field.
10. A mass m suspended by a massless spring of spring constant k . The suspension point is pulled upward with constant acceleration a_0 . Find the Hamiltonian of the system, Hamilton's equation of motion and equation of motion.
11. Obtain Hamilton's equation for a particle of mass m moving in a plane about a fixed point by an inverse square force $-\frac{k}{r^2}$. Hence 1) Obtain the radial equation of motion 2) Show that the angular momentum is constant.
12. Obtain Hamilton's equation for a simple pendulum. Hence obtain an expression for its period.
13. Show that the shortest distance between two points is a straight line.

Long Answer Questions

1. Write the Hamilton's principle for non holonomic systems
2. Obtain Hamilton's equation for a particle of mass moving in a plane about a fixed point by an inverse square force $-\frac{k}{r^2}$. Hence 1) Obtain the radial equation of motion 2) show that the angular momentum is constant.
3. A particle of mass m moves in three dimensions under the action of a central conservative force with potential energy $V(r)$. Then 1) find the Hamiltonian function in spherical polar coordinates 2) Show that ϕ is an ignorable coordinate 3) Obtain Hamilton's equations of motion and 4) express the quantity $l^2 = m^2 r^4 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2)$ in terms of generalized momenta.
4. A bead of mass m slides on a frictionless wire under the influence of gravity. The shape of wire is parabolic and it rotates about the z axis with constant angular velocity ω . Taking $z^2 = a\rho$ as the equation of the parabola, obtain the Hamiltonian of the system. Is $H=E$?
5. Define the Hamiltonian. When is it equal to the total energy of the system? When is it conserved?
6. Obtain Hamilton's equation for the projectile motion of a particle of mass m in the gravitational field. Hence, show that the cyclic coordinate in it is proportional to the time of flight if the point of projection is the origin.
7. Describe Hamiltonian and Hamilton's equation for an ideal spring mass arrangement.
8. Describe the Hamiltonian and Hamilton's equation of motion for charged particle in an electromagnetic field
9. Obtain Hamiltonian equation for a compound pendulum. Obtain an expression for its period. $T = 2\pi \sqrt{\frac{I}{Mgl}}$
10. Show that for relativistic free particle, Hamilton's is given by

$$H(r, p) = \sqrt{p^2 c^2 + m_0^2 c^4} + V(r)$$

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Chapter-3: Canonical Transformations and Poisson's Bracket

Objectives

Short Answer Questions

1. Solve the problem of harmonic oscillator in one dimension by effecting a canonical transformation.
2. What is canonical transformation.
3. If $[F, G]$ be the Poisson-bracket, then prove that $\frac{\partial}{\partial t} [F, G] = \left[\frac{\partial F}{\partial t}, G \right] + \left[F, \frac{\partial G}{\partial t} \right]$
4. Obtain Hamilton's equations for the projectile motion of a particle of mass m in the gravitational field. Hence, show that the cyclic co ordinate in it is proportional to the time of flight if the point of projection is the origin.
5. Find the Poisson bracket of $[L_x, L_y]$, where L_x and L_y are angular momentum components.
6. If $[\Phi, \Psi]$ be the Poisson-bracket, then prove that $\frac{\partial}{\partial t} [\Phi, \Psi] = \left[\frac{\partial \Phi}{\partial t}, \Psi \right] + \left[\Phi, \frac{\partial \Psi}{\partial t} \right]$
7. Using Poisson Bracket, prove that $[L_z, L_x] = L_y$
8. Define Poisson bracket and state its important properties.
9. Show that the transformation

1. $Q = \frac{1}{p}$ and $P = qp^2$

10. Show that the transformation

- a. $p = m\omega q \cot Q$ and $P = \frac{m\omega q^2}{2\sin^2 Q}$ is canonical, and obtain the generator of the transformation.

11. Show that the transformation $Q = \ln\left(\frac{1}{q} \sin p\right)$ and $P = q \cot p$ is canonical. Also obtain the generating function for the transformation.

12. For what values of α and β

- a. $Q = q^\alpha \cos \beta p$ $P = q^\alpha \sin \beta p$

- b. Represent a canonical transformation. Also find the generator of the transformation

13. Show that the following transformation is canonical

- a. $Q = \sqrt{2q} e^\alpha \cos p$ $P = \sqrt{2q} e^{-\alpha}$ α is constant

Long Answer Questions

1. Show that the transformation $Q = \tan^{-1}\left(\frac{\alpha q}{p}\right)$, $P = \frac{1}{2}\alpha q^2\left(1 + \frac{p^2}{\alpha q^2}\right)$ for any constant α is canonical.
 2. Using Poisson Bracket, prove that $Q = \sqrt{[e^{-2q} - p^2]}$ and $P = \cos^{-1}(pe^q)$ is canonical.
 3. Show that the transformation $Q = q \tan p$, $P = \log(\sin p)$ is canonical.
 4. Prove that under canonical transformation (q, p) to (Q, P) Poisson bracket remains invariant.
 5. Using Poisson Bracket, prove that $[L_x, P_y] = P_z$ and $[J_x, p_x] = 0$
 6. Using the Poisson bracket, show that the transformation $q = \sqrt{2P} \sin Q$, $p = \sqrt{2P} \cos Q$ is canonical.
 7. Prove that the transformation $Q = q \cot p$, $P = \log\left(\frac{\sin p}{q}\right)$ is canonical and find the generating function.
 8. For what values of α and β the following transformation is canonical $Q = q^{\alpha-1} \cos \beta p$
 $p = \sqrt{q} \sin \beta p$ also find the generating function.
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Chapter-4: Central Force

Objectives

1. A particle is moving under central force about a fixed centre of force. Choose the correct statement
 - a. The motion of particle is always on a circular path
 - b. Its angular momentum is conserved**
 - c. Its kinetic energy remains constant
 - d. motion of particle takes place in a plane**
2. Two particles of masses m and $2m$, interacting via gravitational force are rotating about common centre of mass with angular velocity ω at fixed distance r . If the particle of mass $2m$ is taken as the origin O
 - a) The force between them can be represented as $F = \mu\omega^2 r$**
 - b) In an inertial frame, fixed at the centre of mass, the origin is at rest
 - c) In the inertial frame, the origin O is moving on a circular path of radius $r/3$**
 - d) In the inertial frame, the particle of mass m is moving on a circular path of radius $r/3$
3. A particle is moving on elliptical path under inverse square law force of the form $F(r) = -\frac{k}{r^2}$.
The eccentricity of the orbit is
 - a) A function of total energy**
 - b) Independent of total energy
 - c) A function of angular momentum**
 - d) Independent of angular momentum
4. The maximum and minimum velocities of a satellite are v_1 and v_2 respectively. The eccentricity of the orbit of the satellite is given by
 - a) $e = \frac{v_1}{v_2}$
 - b) $e = \frac{v_2}{v_1}$
 - c) $e = \frac{v_1 - v_2}{v_1 + v_2}$
 - d) $e = \frac{v_1 + v_2}{v_1 - v_2}$**
5. Rutherford's differential scattering cross section
 - a) Has the dimensions of area**
 - b) Has the dimensions of solid angle
 - c) Is proportional to the square of the kinetic energy of the incident particle
 - d) Is inversely proportional to $\text{cosec}^4\left(\frac{\theta}{2}\right)$, where θ is the scattering angle
6. Consider a comet of mass m moving in a parabolic orbit around the sun. The closest distance between the comet and the sun is b , the mass of the sun is M and the universal gravitational constant is G .
 - 1) The angular momentum of the comet is

- a) $M\sqrt{Gmb}$ b) $b\sqrt{GmM}$ c) $G\sqrt{Mmb}$ d) $M\sqrt{2GMb}$

2) Which one of the following is true for the above system?

- a) **The acceleration of the comet is maximum when it is closest to the sun**
b) The linear momentum of the comet is a constant
c) The comet will return to the solar system after a specified period
d) The kinetic energy of the comet is a constant

7. Consider two satellites A and B revolving around the earth in circular orbits with radii R_A and R_B . Their periods T_A and T_B are 8h and 1h, respectively. The ratio R_A/R_B is equal to

- a) $8^{3/2}$ b) 8 c) 4 d) $8^{1/2}$

8. A satellite in a circular orbit about the earth has a kinetic energy E_k . What is the minimum amount of energy to be added, so that it escape from the earth?

- a) $E_k/4$ b) $E_k/2$ c) E_k d) $2E_k$

9. Two planets of masses M_1 and M_2 have satellites of masses m_1 and m_2 respectively, revolving around them at the same radius r . The period of the first satellite (of mass m_1) is twice as that of the second. Which one of the following relation is correct?

- a) $4M_1=M_2$ b) $2M_1= M_2$ c) $M_1 = 2M_2$ d) $m_1M_1 = m_2M_2$

10. Let R_s and R_m be the distances of the geostationary satellite and moon from the centre of the earth. Then, R_m/R_s is approximately

- a) $(29)^{1/2}$ b) $(29)^{2/3}$ c) 29 d) $(29)^{3/2}$

Short Answer Questions

1. State the Kepler's first law of planetary motion
2. State and prove Kepler's second law of planetary motion
3. State the Kepler's third law of planetary motion
4. Define elliptical orbit
5. Calculate the reduced mass of H₂ molecule. Assume the mass of H atom is M.
(Ans. M/2)
6. Define hyperbolic orbit
7. Define parabolic orbit
8. Which force is required to obtain circular motion of the particle around the centre of the force
9. Show that for a particle moving through inverse square law forces, areal velocity remains constant.
10. Prove that total energy of a particle moving through a central force is a constant of motion
11. State any two gyroscopic forces. Prove that gyroscopic forces doesn't consume power.
12. Explain geosynchronous and geostationary orbits. State the uses of artificial satellites.
13. Write coriolis force for
 - a. River flow on the surface of the earth
 - b. Formation of cyclones
14. State and prove virial theorem.
15. Show that angular momentum of a particle moving in a central force field is conserved.
16. A particle moves with velocity in an elliptical path in an inverse fixed. Prove that
$$v^2 = \frac{k}{\mu} \left[\frac{2}{r} - \frac{1}{a} \right]$$
17. A particle moving in a force field has the equation of its orbit $r = a\theta$. Find the laws of force.
18. If the eccentricity of a planets orbit is e, find the ratio of maximum to minimum speeds of the planet in its orbit.
19. If the eccentricity of a planets orbit is e, find the ratio of maximum to minimum speed of the planet in its orbit.
20. A planet moving in an elliptical orbit has its smallest speeds v_{min} and v_{max} respectively.

Show that the eccentricity of orbit is

$$\frac{v_{max} - v_{min}}{v_{max} + v_{min}}$$

21. A particle of mass m moves under a central force field given by $= -\frac{kr^2}{r^4}$. If E is the energy

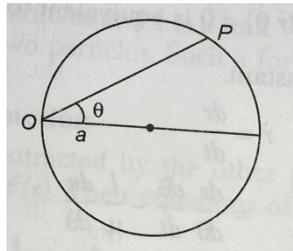
of particle then, show that its speed is given by $v = \sqrt{\frac{K}{mr^2} + \frac{2E}{m}}$

22. Consider a particle moving in a central field of force. If we consider the radial motion only then

- a) What is the effective potential in which the radial motion occurs?
- b) Calculate the angular frequency for circular orbit, if the central potential is $1/2kr^2$

Long Answer Questions

1. A particle describes circular orbit given by $r = 2a \cos\theta$, under the influence of an attractive central force directed towards a point on the circle. Show that force varies as the inverse of fifth power of distance.
2. Derive equation of motion for a particle moving under central force. What is the form of equation, when the particle is moving under an attractive inverse square law force $F = -\frac{k}{r^2}$.
3. For the equation of the orbit given by the conic $r = \frac{l}{1 + e \cos\theta}$. Where l and e are constants. Find the law of force and show that it follows an inverse square law of force.
4. A particle describes a circular path under the influence of an attractive central force directed towards a fixed point on the circle. Find the law of force.



5. Show that the velocity of a planet undergoing an elliptical path having semi major axis a , at a point distant r from the centre of force is given by $v^2 = \frac{c}{\mu} \left(\frac{2}{r} - \frac{1}{a} \right)$, where the force law is $f(r) = -\frac{c}{r^2} \hat{r}$ and μ is reduced mass.
6. If v_1 and v_2 are the velocities at the end of any diameter passing through the centre of the elliptical path described by a particle, then prove that $v_1 v_2$ is a constant and equals to $\frac{c}{\mu a}$
7. A particle is projected from the earth's surface with a speed v , and describes an elliptical orbit.
23. A particle of mass m , moves under the action of a central force whose potential is $V(r) = kmr^3$ ($k > 0$); then
 - a) For what energy and angular momentum will the orbit be a circle of radius, a , about the origin?
 - b) Calculate the period of circular motion
 - c) If the particle is slightly distributed from the circular motion, what is the period of small radial oscillations about $r = a$.
8. Show that the velocity of planet undergoing an elliptical path having semi major axis a , at a point distant r from the centre of force is given by $v^2 = \frac{c}{\mu} \left(\frac{2}{r} - \frac{1}{a} \right)$ where the force law is

$f(r) = -\frac{C}{r^2} \dot{r}$ and μ is reduced mass. Hence show that this speed is the same as it would have been if it had fallen from a point distant equal to the length of the major axis to that point.