Anekant Education Society's

TuljaramChaturchand College, Baramati

Department of Mathematics

Class: M.Sc –I

Question Bank

Title of Paper : Advanced Calculus **Sub Code:**MAT4102

1) State True or False with justification:-

If $f'(x;y)=0$ for every x in some n-ball $B(a)$ for every vector y then f is constant on (a) .

2) State sufficient condition for differentiability.

3) Define line integral.

4) Write any two applications of chain rule.

5) Define surface integral and explain the terms involves in it.

6) Write the matrix form of the chain rule.

7) Is composition of any two functions continuous?

8) Define derivative of a scalar field with respect to a vector.

9) Give any two properties of gradient.

10) Give an example of scalar field f such that $f'(x;y)>0$ for a fixed vector $\bar{y}$ and every vector $\bar{x}$.

11) State any two examples of convex set.

12) Find the gradient vector of f$(x,y)= e^x cosy$ at each point at which it exist.

13) Let f and g denote scalar field that are differentiable on an open set S, then derive grad f =0 if f is constant on S.

14) Write the formula for grad (f.g)

15) Give applications of line integral.

16) Define open connected set.

17) State True or False with justification:-

$\oint_c \bar{f} \neq 0$ for a particular close curve c then $\bar{f}$ is gradient.

18) Show that $\bar{f}\,(x,y)=xi+xyj$ is not a gradient.

19)Determine whether a vector field $\bar{f}\,(x,y)=3x^2yi+x^3yj$ is a gradient on any open subset of R^2?

20) If $\bar{y}=\bar{0}$ then calculate $f'(\bar{a};\bar{0})$.

21) Evaluate derivative of linear transformation.

22) Compute $f'(\bar{a};\bar{y})$,if f(x) =$\|\bar{x}\|^2$ for every x in R^n

23) Prove that every open ball is convex.

24) Give any 5 examples of convex sets.

25) Show that if scalar field f is differentiable at $\bar{a}$ then f is continuous at$\bar{a}$.

26) Find gradient vector of $f(x,y) = x^2 + y^2 sinxy$,at each point for it exist.

27) Make a sketch to describe level set of$f(x,y) = x^2 + y^2$, where c = 0,1.

28)Draw level curve of f(x,y) = e^{xy} ,at c= e^{-2} ,e^{-1} .

29) Draw set $x^2 + y^2 < 1$.

30) Make a sketch to describe set x > 0 and y<0.

31) Find divergence of $f(x,y,z) = xi + yj + zk$.

32)Find curl of $f(x,y,z) = xi + yj + zk$.

33) find Jacobian matrix of $f(x,y,z) = xy^2z^2i + z^2sinyj + x^2k$.

34) State any two properties of curl.

35) State any two properties of divergence.

36) State Stokes Theorem.

37) Write the applications of Stokes Theorem.

38) Write the formula for surface area.

39) State Implicit Function Theorem.

40) State Inverse Function Theorem.

41) Sketch set simply connected and convex.

42) Sketch set simply connected but not convex.

43) Give some applications of green's theorem.

44) State green's theorem for a plane region .

45) Is every function defined on set has a lower integral?

46) Find directional derivative of scalar field $\bar{f}\,(x,y) = x^2 - 3xy$ along y= x^2-x+2 at point (1,2).

47) If $f(x,y) = \int_0^{\sqrt{xy}} e^{-t^2}\, dt$ for x>0 , y>0 then compute $\frac{\partial f}{\partial x}$ in terms of x and y.

48) Calculate line integral of vector field $\overline{f}$ (x,y) =(x^2 -2xy)i+(y^2-3xy)j from (-1,1) to (1,1) along parabola $y=x^2$.

49)Calculate line integral of vector field $\overline{f}$ (x,y ,z) =xi+yj+(xz-y)k from (0,0,0) to (1,2,4) along a line segment.

50) Let $\bar{f}(x,y) = P(x,y)i + Q(x,y)$j where partial derivative $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ are continuous on an open set S. If $\bar{f}$ is gradient of some potential ϕ then prove that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ at each point of set S.

Answer in One sentence

51) What is open ball in R^1.

52) Write interior of [0,1].

53) Determine the set of points of continuity of $f(x,y) = \frac{1}{y} cos(x^2)$.

54) Find set of discontinuity of $(x,y) = x^4 + y^4 - 4x^2y^2$.

55) Compute $\frac{\partial^2 f}{\partial x \partial y}$ [log (x^2+y^2)] , (x,y) ≠(0,0).

56) Compute $\frac{\partial f}{\partial x}$ [tan($\frac{x^2}{y}$)].

57) Find $D_1\, f(\bar{0})$of f(x,y) = $\frac{xy^2}{x^2+y^4}$; if x≠0, and 0 if x=0.

58) State sufficient condition for equality of mixed partial derivative.

59) State basic properties of line integral.

60) Define line integral w.r.to. arc length.

61) State second fundamental theorem of calculus for line integral.

62) What is closed path C such that $\oint_C \bar{F} \neq 0$ for $\overline{f}\,(x,y) = yi - xj$.

63) Find closed path C such that $\oint_C \bar{F} \neq 0$ for $\overline{f}\,(x,y) = yi + (xy - x)j$.

64) Give necessary and sufficient condition for vector field to be gradient.

65) Is $\overline{f}\,(x,y) = yi - xj\ \ gradient$?

66) Write vector equation of surface.

Short Notes on

67) Write short note on cylindrical co-ordinate system.

68) Write a short note on double integral of step function.

Long questions

69)Compute the area of the region cut from the plane x+y+z = a by the cylinder $x^2+y^2=a^2$.

70) Prove second fundamental theorem for line integral.

71) Find the directional derivative of scalar field $f(x,y,z) = x^2 + 2y^2 + 3z^2$ at (1,1,0) in the direction $\bar{\imath} - \bar{\jmath}+2\bar{k}$.

72) If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and if one dimensional limit $\lim_{x\to a} f(x,y)$ *and* $\lim_{y\to b} f(x,y)$ both exist,

Then prove that$\lim_{x\to a}[\lim_{y\to b} f(x,y)] = \lim_{y\to b}[\lim_{x\to a} f(x,y)]$ = L.

73) Show that a line integral remains unchanged under a change of parameter that preserves orientation.

74) If $\bar{f}(x,y) = \frac{x^2y^2}{x^2y^2+(x-y)^2}$ *where* $x^2y^2 + (x-y)^2 \neq 0$. Then show that

$\lim_{x\to 0}[\lim_{y\to 0} f(x,y)] = \lim_{y\to 0}[\lim_{x\to 0} f(x,y)]$ = 0.

75) Compute all first order partial derivatives of $f(x,y) = x^2 + y^2 cosxy$ on the given scalar field.

76) Let $f(x,y) = arc\tan\left(\frac{y}{x}\right)$, $x \neq 0$ then show that $D_2(D_1F) = D_1(D_2F)$.

77) Let S be a set of all (x,y) ≠ (0,0) in R^2, f be the vector field defined on set S by the equation

$\bar{f}(x,y) = \frac{y\bar{\imath}}{x^2+y^2} + \frac{x\bar{\jmath}}{x^2+y^2}$ then show that $D_1 f_2 = D_2 f_1$ everywhere on S.

78) Define double integral of a bounded function over a rectangle. prove that every function f which is bounded on a rectangle Q has a lower integral $\underline{I}(F)$ and an upper integral $\bar{I}$(F) .

79) Define area of parametric surface and find area of hemisphere $x^2 + y^2 + z^2 = a^2, z > 0$.

80) Calculate line integral of vector field $\overline{f}$ (x, y) =(x+y)i+(x-y)j once around the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ in a counterclockwise direction.

81) State and prove Gauss divergence theorem.

82) If Q = [-1,1] × [0,2] then find $\iint_Q \sqrt{|y-x^2|}\ dxdy$.

83) If $\bar{f}: R^2 \to R^2$ and $\bar{g}: R^2 \to R^2$ be the two vector fields defined as follows:

$\bar{f}$ (x, y) $= e^{x+2y} i + \sin(y+2x)\, j$, $\bar{g}$ (x, y,w) =(u+$2v^2 + 3w^3$)i +$(2v-u^2)\, j$,compute each of the Jacobian matrices$D\bar{F}(x,y)$ and $D\bar{g}(u,v,w)$.

84) $\bar{f}$ and $\bar{g}$ defined above calculate composition $\bar{h}\,(u,v,w) = \bar{f}\,[\bar{g}(u,v,w)]$.

85) $\bar{f}$ and $\bar{g}$ defined above calculate Jacobian matrix $D\bar{h}\,(1,-1,1)$.

86) Compute mass M of one coil of a spring having the shape of helix whose vector equation is$\bar{\alpha}$ (t) = acost$\bar{\imath}$ +asint $\bar{\jmath}$+bt $\bar{k}$ if the density at (x,y,z) is $x^2+y^2+z^2$.

89) state and prove Green's theorem for a plane regions bounded by piecewise smooth Jordan curve.

90) Transform the integral to polar coordinates and compute its value

$$\int_0^{2a} \cdot\Big[\int_0^{\sqrt{2ax-x^2}} (x^2+y^2)dy\Big]dx.$$

91)If the component of $\bar{F}$ has all mixed partial derivatives continuous then prove

Curl(Curl $\bar{F}) = grad(div\bar{F}) - \nabla^2\bar{F}$.

92) Use green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$ where C is the square with vertices (0,0),(2,0),(2,2),(0,2).

93) Use green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$ where C is the square with vertices (±1,±1).

94) Use green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$ where C is the square with vertices (±2,0),(0,±2).

95) Use green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$ where C is the circle with radius 2 and centre at the origin.

96) Use green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$ where C has the vector equation

$\bar{\alpha}$(t)=$2cos^3 ti + 2sin^3 tj$, where $0 \le t \le 2\pi$.

97) State and prove chain rule for derivatives of vector field.

98) Define fundamental vector product ,show that fundamental vector product is normal to the surface.

99) State Stoke's theorem and show that the surface integral which appears in Stoke's theorem in terms of the curl of a vector field.

100) Determine jacobian matrix and compute the curl and divergence

of $\bar{F}(x, y, z) = xy^2z^2\bar{\imath} + z^2 siny\bar{\jmath} + x^2\ \bar{k}$.